MATH 3551 A — Abstract Algebra I
Practice Midterm 1 — Version 2

Taylor Dupuy, University of Vermont • Fall 2026

For practice — Midterm 1 is Monday, October 12, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): ____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

Version 2 — September 28, 2026. Generated by GPT-5.6 Sol in the Codex desktop harness. Please notify Professor Dupuy if something looks off.

What changed from Version 1. The coverage now stops before §1.7. One true/false item and Problem 4 were replaced because §2.2 and Chapter 3 are not on Midterm 1. Selected material from §§0.1–0.3 has been added to make the foundations easier to practice, and that material is included on the exam. All other questions are unchanged. Version 1 remains available on the course webpage so that the changes can be compared directly.

This practice exam targets about thirty minutes of work in a fifty-minute period. Please report any error, ambiguity, out-of-coverage question, or item that takes much longer than its point value suggests.

Problem 1. (30 points) **True or false; prove or disprove.**

Circle **T** or **F**, and then give a one-line reason: a theorem, a short computation, or an explicit counterexample. Five points each: two for the circle, three for the reason.

(a) T F There is an element of order 18 in the symmetric group S_9.

(b) T F The map $\varphi\colon \mathbb{Q}^\times \to \mathbb{Q}^\times$ defined by $\varphi(x) = |x|$ is a group homomorphism.

(c) T F The class $\overline{6}$ has a multiplicative inverse in $\mathbb{Z}/15\mathbb{Z}$.

(d) T F The group $(\mathbb{Z}/15\mathbb{Z})^\times$ is cyclic.

(e) T F The number of 3-cycles in S_6 is 40.

(f) T F The groups $\mathbb{Z}/4\mathbb{Z}$ and $(\mathbb{Z}/8\mathbb{Z})^\times$ are isomorphic.

Problem 2. (25 points) **Inside $\mathbb{Z}/72\mathbb{Z}$.**

Work in the additive group $\mathbb{Z}/72\mathbb{Z}$.

(a) (4 points) Find the order of $\overline{30}$.

(b) (5 points) List the elements of the subgroup $\langle\overline{30}\rangle$.

(c) (4 points) How many generators does $\mathbb{Z}/72\mathbb{Z}$ have?

(d) (7 points) Determine every k with $0 \leq k < 72$ for which $\langle\overline{k}\rangle = \langle\overline{18}\rangle$.

Problem 3. (25 points) **Two permutations.**

In S_9 let

$$\sigma = (1\,4\,7)(2\,5)(3\,6\,8\,9), \qquad \tau = (1\,2\,3\,4\,5).$$

(Permutations act on the left, so $\sigma\tau$ means: first apply τ, then apply σ.)

(a) (4 points) What is $|\sigma|$?

(b) (6 points) Find the cycle decomposition of $\sigma\tau$, and its order.

(c) (3 points) For which k with $0 \le k < 12$ does σ^k fix the point 3?

(d) (2 points) Give the cycle decomposition of σ^{-1}.

Problem 4. (20 points) **Subgroups of the quaternion group.**

Let $G = Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$, with the usual relations $i^2 = j^2 = k^2 = -1$, $ij = k$, and $ji = -k$.

(a) (4 points) Determine the order of every element of G.

(b) (5 points) List the elements of $\langle i \rangle$, $\langle j \rangle$, and $\langle i, j \rangle$.

(c) (6 points) List every subgroup of Q_8.

(d) (5 points) Explain why Q_8 is not isomorphic to D_8.