MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version A

Taylor Dupuy, University of Vermont • Fall 2026

Practice for November 6, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): _____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

About this practice exam. Revised with Codex on October 26, 2026. It targets about twenty minutes of work in the full fifty-minute period and covers §1.7, §2.2, and §§3.1–3.3. Please report errors, ambiguity, out-of-coverage material, or unexpectedly long items to Professor Dupuy.

Problem 1. (20 points) **True or false; prove or disprove.**

Circle T or F and give a one-line reason. Each item is worth 5 points (2 for the answer, 3 for the reason).

(a) **T** **F** Every quotient of an abelian group is abelian.

(b) **T** **F** For every subgroup $H \leq G$, one has $N_G(H) = G$.

(c) **T** **F** The kernel of a group homomorphism is a normal subgroup.

(d) **T** **F** The subgroup $\langle \overline{4} \rangle$ of $\mathbb{Z}/12\mathbb{Z}$ has order 4.

Problem 2. (30 points) **A quotient of Q_8.**

Let $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ and $N = \{1, -1\}$.

(a) (12 points) Show that $N \trianglelefteq Q_8$ and list the cosets of N.

(b) (18 points) Identify Q_8/N up to isomorphism and justify why it is not cyclic.

Problem 3. (30 points) **A normalizer in S_4.**

Let $H = \langle (1\,2\,3) \rangle \leq S_4$.

(a) (15 points) Determine $C_{S_4}(H)$ and justify your answer.

(b) (15 points) Determine $N_{S_4}(H)$ and decide whether $H \trianglelefteq S_4$.

Problem 4. (20 points) **The Second Isomorphism Theorem in D_{10}.**

Let $G = D_{10} = \langle r, s \mid r^5 = s^2 = 1,\ rs = sr^{-1} \rangle$, $N = \langle r \rangle$, and $H = \langle s \rangle$.

(a) (10 points) Show that $N \trianglelefteq G$ and find $H \cap N$ and HN.

(b) (10 points) Identify $H/(H \cap N)$ and HN/N, and explain the isomorphism between them.

End of practice exam. Total: $20 + 30 + 30 + 20 = 100$ points.