MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version B — SOLUTIONS

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Fall 2026 • Total: 100 points

Problem 1. (20 points) **True or false; prove or disprove.**

(a) FALSE. The cyclic group $\mathbb{Z}/6\mathbb{Z}$ has a subgroup $\langle\overline{2}\rangle$ of order 3, smaller than 6.

(b) TRUE. A subgroup of index 2 has only one nontrivial left coset and one nontrivial right coset, so the left and right cosets agree; thus it is normal.

(c) TRUE. The projection sends g to gN; it sends g to the identity coset exactly when $g \in N$.

(d) FALSE. The natural action of S_4 on four points has an orbit of size 4, but S_4 has order 24.

Grading note. Award 2 points for the correct answer and 3 for a specific reason.

Problem 2. (30 points) **A homomorphism from D_{12}.**

(a) The image of r has order 3, so its sixth power is 1; the image of s has order 2. Conjugation by $(1\,2)$ inverts $(1\,2\,3)$, so $\varphi(r)\varphi(s) = \varphi(s)\varphi(r)^{-1}$. Thus the relations hold. A 3-cycle and a transposition generate S_3, so φ is onto. (15)

(b) Since φ is onto, its kernel has order $12/6 = 2$. The element r^3 maps to 1 and has order 2, so $\ker\varphi = \langle r^3\rangle$. The First Isomorphism Theorem gives $D_{12}/\langle r^3\rangle \cong S_3$. (15)

Grading note. In (a), check the inversion relation and surjectivity. In (b), use the kernel order and exhibit r^3.

Problem 3. (30 points) **Two subgroups of $\mathbb{Z}/24\mathbb{Z}$.**

(a) $H = \{\overline{0}, \overline{8}, \overline{16}\}$ and $N = \{\overline{0}, \overline{6}, \overline{12}, \overline{18}\}$. Thus $H \cap N = \{\overline{0}\}$ and $H + N = \langle\overline{2}\rangle$, the order-12 subgroup of G. (15)

(b) $H/(H \cap N) = H \cong \mathbb{Z}/3\mathbb{Z}$. Also $(H + N)/N$ has order $12/4 = 3$ and is cyclic. The map $h + (H \cap N) \mapsto h + N$ is onto and has trivial kernel, giving the claimed isomorphism. (15)

Grading note. Require the subgroup lists and the intersection in (a). In (b), explain the actual induced map, not merely that both sides have order three.

Problem 4. (20 points) **A quotient of D_{12}.**

(a) Conjugation by r fixes $\langle r^2 \rangle$, and conjugation by s sends r^2 to r^{-2}, so $N \trianglelefteq D_{12}$. In the quotient, $r^2 = 1$, while $s^2 = 1$ and $srs = r^{-1} = r$ (as r has order 2 there). Thus the quotient is $C_2 \times C_2$. (10)

(b) The subgroups containing N correspond to the five subgroups of the Klein four quotient: N maps to 1; $\langle r \rangle$ maps to $\langle rN \rangle$; $\langle r^2, s \rangle$ maps to $\langle sN \rangle$; $\langle r^2, rs \rangle$ maps to $\langle rsN \rangle$; and D_{12} maps to the whole quotient. These are all five by the Fourth Isomorphism Theorem. (10)

Grading note. The quotient is the Klein four-group and has exactly three order-two subgroups. Their three distinct preimages, along with N and D_{12}, complete the list.