MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version B

Taylor Dupuy, University of Vermont • Fall 2026

Practice for November 6, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): _____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

About this practice exam. Revised with Codex on October 26, 2026. It targets about twenty minutes of work in the full fifty-minute period and covers §1.7, §2.2, and §§3.1–3.3. Please report errors, ambiguity, out-of-coverage material, or unexpectedly long items to Professor Dupuy.

Problem 1. (20 points) **True or false; prove or disprove.**

Circle T or F and give a one-line reason. Each item is worth 5 points (2 for the answer, 3 for the reason).

(a) **T** **F** Every subgroup of a cyclic group has the same order as the group.

(b) **T** **F** Every subgroup of index 2 is normal.

(c) **T** **F** If $N \trianglelefteq G$, the natural map $G \to G/N$ has kernel N.

(d) **T** **F** A group action with an orbit of size 4 must come from a group of order 4.

Problem 2. (30 points) **A homomorphism from D_{12}.**

Let $G = D_{12} = \langle r, s \mid r^6 = s^2 = 1,\ rs = sr^{-1} \rangle$. Define $\varphi : G \to S_3$ by $\varphi(r) = (1\,2\,3)$ and $\varphi(s) = (1\,2)$.

(a) (15 points) Check that these assignments respect the defining relations and show that φ is onto.

(b) (15 points) Find $\ker \varphi$ and identify $G/\ker \varphi$ using the First Isomorphism Theorem.

Problem 3. (30 points) **Two subgroups of $\mathbb{Z}/24\mathbb{Z}$.**

Work additively in $G = \mathbb{Z}/24\mathbb{Z}$; let $H = \langle \overline{8} \rangle$ and $N = \langle \overline{6} \rangle$.

(a) (15 points) List H, N, $H \cap N$, and $H + N$.

(b) (15 points) Identify $H/(H \cap N)$ and $(H+N)/N$ and explain the Second Isomorphism Theorem map between them.

Problem 4. (20 points) **A quotient of D_{12}.**

Let $D_{12} = \langle r, s \mid r^6 = s^2 = 1,\ rs = sr^{-1} \rangle$.

(a) (10 points) Show that $N = \langle r^2 \rangle$ is normal and determine D_{12}/N.

(b) (10 points) List all subgroups of D_{12} containing N and identify their images in D_{12}/N.

End of practice exam. Total: $20 + 30 + 30 + 20 = 100$ points.