MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version C

Taylor Dupuy, University of Vermont • Fall 2026

Practice for November 6, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): _____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

About this practice exam. Revised with Codex on October 26, 2026. It targets about twenty minutes of work in the full fifty-minute period and covers §1.7, §2.2, and §§3.1–3.3. Please report errors, ambiguity, out-of-coverage material, or unexpectedly long items to Professor Dupuy.

Problem 1. (20 points) **True or false; prove or disprove.**

Circle T or F and give a one-line reason. Each item is worth 5 points (2 for the answer, 3 for the reason).

(a) **T** **F** If $\varphi : G \to K$ is onto and $N \trianglelefteq G$, then $\varphi(N) \trianglelefteq K$.

(b) **T** **F** Every quotient of a cyclic group is cyclic.

(c) **T** **F** Every group of order 6 is cyclic.

(d) **T** **F** The kernel of a group action is the set of elements fixing every point.

Problem 2. (30 points) **A matrix-group quotient.**

Work over $\mathbb{F}_3$. Let $G = \mathrm{GL}_2(\mathbb{F}_3)$ and $N = \mathrm{SL}_2(\mathbb{F}_3) = \ker(\det)$.

(a) (12 points) Count the elements of G by choosing the two columns of an invertible matrix.

(b) (18 points) Show that $\det : G \to \mathbb{F}_3^\times$ is onto. Determine $|N|$ and identify G/N using the First Isomorphism Theorem.

Problem 3. (30 points) **Subgroups and quotients of $\mathbb{Z}/18\mathbb{Z}$.**

Write $G = \mathbb{Z}/18\mathbb{Z}$ additively.

(a) (15 points) List all subgroups of G, giving a generator and order for each.

(b) (15 points) For each subgroup H of order 3 or 6, identify the quotient G/H up to isomorphism.

Problem 4. (20 points) **A quotient of a quotient of Q_8.**

Let $G = Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$, $N = \{1, -1\}$, and $K = \langle i \rangle$.

(a) (10 points) Identify G/N, K/N, and G/K up to isomorphism.

(b) (10 points) Describe the Third Isomorphism Theorem map $(G/N)/(K/N) \to G/K$ in this example.

End of practice exam. Total: $20 + 30 + 30 + 20 = 100$ points.