MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version D — SOLUTIONS

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Problem 1. (20 points) **True or false; prove or disprove.**

(a) FALSE. S_3/A_3 is cyclic, but S_3 is not cyclic.

(b) TRUE. By orbit–stabilizer each stabilizer has order $|G|/|Gx|$, and transitivity makes all orbits have the same size.

(c) TRUE. This is the First Isomorphism Theorem: the induced map $g\ker\varphi \mapsto \varphi(g)$ is bijective when φ is onto.

(d) FALSE. In S_3, the subgroup $\langle(1\,2)\rangle$ has index 3 but is not normal: $(1\,3)(1\,2)(1\,3) = (2\,3)$ is not in it.

Grading note. Award 2 points for the circle and 3 for a specific reason. The counterexample in (d) is the nonnormal index-three subgroup of S_3.

Problem 2. (30 points) **A subgroup of D_8.**

(a) r^2 is central and has order 2, while s has order 2, so $H = \{1, r^2, s, r^2s\}$ has order 4. It has index 2 in G, hence is normal. (12)

(b) Every element centralizes r^2. A rotation r^i commutes with s only when $r^{2i} = 1$, so $i = 0, 2$; the same condition applies to a reflection sr^i. Thus $C_G(H) = C_G(s) = \{1, r^2, s, r^2s\} = H$. Since H is normal, $N_G(H) = G$. (18)

Grading note. Distinguish the four-element centralizer from the full normalizer. Index two proves normality without a long conjugation check.

Problem 3. (30 points) **Subgroups above N in $\mathbb{Z}/60\mathbb{Z}$.**

(a) $N = \langle\overline{10}\rangle$ has order 6. A cyclic group has a unique subgroup of each divisor order, and those containing N have orders $6, 12, 30, 60$: respectively $\langle\overline{10}\rangle$, $\langle\overline{5}\rangle$, $\langle\overline{2}\rangle$, and $G = \langle\overline{1}\rangle$. (15)

(b) G/N is cyclic of order $60/6 = 10$, hence $G/N \cong \mathbb{Z}/10\mathbb{Z}$. The four listed subgroups map to the unique subgroups of G/N of orders $1, 2, 5, 10$, respectively, because the quotient order is $|H|/6$. These are all subgroups of the cyclic order-10 quotient. (15)

Grading note. Include N and G in (a). In (b), match quotient subgroup orders $1, 2, 5, 10$ in the same order as their preimages.

Problem 4. (20 points) **The Second Isomorphism Theorem in D_8.**

(a) $N = \langle r^2 \rangle$ is central, hence normal. Since s commutes with r^2, $HN = \{1, r^2, s, r^2 s\}$, while $H \cap N = \{1\}$. Thus HN/N has order 2 and is cyclic. (10)

(b) $H/(H \cap N) = H/\{1\} \cong C_2$. The map induced by inclusion, $h(H \cap N) \mapsto hN$, is an isomorphism onto HN/N; both sides have order 2. (10)

Grading note. Include the normality of N and identify both sides of the isomorphism. The induced map is well-defined because the intersection is the kernel.