MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version D

Taylor Dupuy, University of Vermont • Fall 2026

Practice for November 6, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): _____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

About this practice exam. Revised with Codex on October 26, 2026. It targets about twenty minutes of work in the full fifty-minute period and covers §1.7, §2.2, and §§3.1–3.3. Please report errors, ambiguity, out-of-coverage material, or unexpectedly long items to Professor Dupuy.

Problem 1. (20 points) **True or false; prove or disprove.**

Circle T or F and give a one-line reason. Each item is worth 5 points (2 for the answer, 3 for the reason).

(a) **T** **F** If $N \trianglelefteq G$ and G/N is cyclic, then G is cyclic.

(b) **T** **F** In a transitive action of a finite group, all point stabilizers have the same order.

(c) **T** **F** If $\varphi : G \to H$ is onto, then $G/\ker\varphi \cong H$.

(d) **T** **F** Every subgroup of index 3 in a finite group is normal.

Problem 2. (30 points) **A subgroup of D_8.**

Let $G = D_8 = \langle r, s \mid r^4 = s^2 = 1,\ rs = sr^{-1} \rangle$ and $H = \langle r^2, s \rangle$.

(a) (12 points) List the elements of H and prove $H \trianglelefteq G$.

(b) (18 points) Determine $C_G(H)$ and $N_G(H)$, with justification.

Problem 3. (30 points) **Subgroups above N in $\mathbb{Z}/60\mathbb{Z}$.**

Work additively in $G = \mathbb{Z}/60\mathbb{Z}$. Let $N = \langle \overline{10} \rangle$.

(a) (15 points) List every subgroup of G containing N, giving a generator and order for each.

(b) (15 points) Identify G/N and match those subgroups with all its subgroups using the Fourth Isomorphism Theorem.

Problem 4. (20 points) **The Second Isomorphism Theorem in D_8.**

Let $D_8 = \langle r, s \mid r^4 = s^2 = 1,\ rs = sr^{-1} \rangle$, $N = \langle r^2 \rangle$, and $H = \langle s \rangle$.

(a) (10 points) Verify that $N \trianglelefteq D_8$, compute HN and $H \cap N$, and identify HN/N.

(b) (10 points) Verify the Second Isomorphism Theorem isomorphism $HN/N \cong H/(H \cap N)$ for these subgroups.

End of practice exam. Total: $20 + 30 + 30 + 20 = 100$ points.