MATH 3551 A — Abstract Algebra I
Practice Midterm 2 — Version 2

Taylor Dupuy, University of Vermont • Fall 2026

For practice — Midterm 2 is Friday, November 6, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): ____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

Version 2 — September 28, 2026. Generated by GPT-5.6 Sol in the Codex desktop harness. Please notify Professor Dupuy if something looks off.

What changed from Version 1. The one-week coverage freeze moves §4.4 to Midterm 3. The automorphism true/false item was replaced by a group-action item; every other question is unchanged. Material deferred from Midterm 1 — §1.7, §2.2, and §§3.1–3.2 — is included in this coverage window. Version 1 remains available on the course webpage so that the changes can be compared directly.

This practice exam targets about thirty minutes of work in a fifty-minute period. Please report any error, ambiguity, out-of-coverage question, or item that takes much longer than its point value suggests.

Problem 1. (30 points) **True or false; prove or disprove.**

Circle **T** or **F**, and then give a one-line reason: a theorem, a short computation, or an explicit counterexample. Five points each: two for the circle, three for the reason.

(a) T F If $H \le G$ and $N \trianglelefteq G$, then HN is a subgroup of G.

(b) T F If $K \trianglelefteq H$ and $H \trianglelefteq G$, then $K \trianglelefteq G$.

(c) T F The symmetric group S_5 has exactly seven conjugacy classes.

(d) T F If a finite group G acts faithfully on a set with five elements, then $|G|$ divides 120.

(e) T F If $|G| = p^3$ for a prime p, then $Z(G) \neq 1$.

(f) T F The alternating group A_4 has a normal subgroup of order 4.

Problem 2. (25 points) **An action of A_4.**

Let $G = A_4$ act on the set $A = \{P_1, P_2, P_3\}$ of the three ways of splitting $\{1, 2, 3, 4\}$ into two unordered pairs:

$$P_1 = \{\{1,2\},\{3,4\}\}, \quad P_2 = \{\{1,3\},\{2,4\}\}, \quad P_3 = \{\{1,4\},\{2,3\}\},$$

where σ acts on a pair by $\sigma \cdot \{i, j\} = \{\sigma(i), \sigma(j)\}$ and on a splitting by acting on each of its two pairs.

(a) (6 points) Determine the permutations of A induced by $(1\,2\,3)$ and by $(1\,2)(3\,4)$.

(b) (6 points) Let $\varphi\colon A_4 \to S_3$ be the resulting permutation representation. Determine $\ker\varphi$ and $\operatorname{im}\varphi$, with proof.

(c) (4 points) Deduce that $V_4 = \{1, (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$ is a normal subgroup of A_4 and identify the quotient A_4/V_4.

(d) (4 points) Determine the stabilizer of P_1 in A_4 and verify the orbit–stabilizer relation.

Problem 3. (25 points) **Conjugacy in S_7.**

(a) (6 points) Determine the number of elements of order 4 in S_7.

(b) (5 points) Determine the size of the conjugacy class of $(1\,2\,3\,4)(5\,6)$ in S_7.

(c) (4 points) Determine the order of $C_{S_7}\big((1\,2\,3\,4)(5\,6)\big)$ and describe its elements explicitly.

Problem 4. (20 points) **Composition series.**

(a) (3 points) Define *composition series* and *composition factors*.

(b) (5 points) Give a composition series for $D_{12} = \langle r, s \mid r^6 = s^2 = 1,\ rs = sr^{-1} \rangle$ and list its composition factors, in order.

(c) (5 points) Give a second composition series for D_{12}, different from the one in (b), in which the composition factors appear in a *different order*. What does the Jordan–Hölder Theorem predict about the two lists?

(d) (2 points) Is D_{12} solvable? Justify your answer from (b).

End of practice exam. Total: $30 + 20 + 15 + 20 + 15 = 100$ points.