MATH 3551 A — Abstract Algebra I
Practice Midterm 3 — Version 2

Taylor Dupuy, University of Vermont • Fall 2026

For practice — Midterm 3 is Monday, November 30, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): ____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

Version 2 — September 28, 2026. Generated by GPT-5.6 Sol in the Codex desktop harness. Please notify Professor Dupuy if something looks off.

What changed from Version 1. The one-week coverage freeze moves §4.4 from Midterm 2 to Midterm 3. One true/false item was replaced by an automorphism item; every other question is unchanged. Version 1 remains available on the course webpage so that the changes can be compared directly.

This practice exam targets about thirty minutes of work in a fifty-minute period. Please report any error, ambiguity, out-of-coverage question, or item that takes much longer than its point value suggests.

Problem 1. (30 points) **True or false; prove or disprove.**

Circle **T** or **F**, and then give a one-line reason: a theorem, a short computation, or an explicit counterexample. Five points each: two for the circle, three for the reason.

(a) **T** **F** Every group of order 35 is cyclic.

(b) **T** **F** There is a simple group of order 351.

(c) **T** **F** For a prime p there are exactly six abelian groups of order p^5 up to isomorphism.

(d) **T** **F** $\mathbb{Z}/12\mathbb{Z} \times \mathbb{Z}/18\mathbb{Z} \cong \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/36\mathbb{Z}$.

(e) **T** **F** The symmetric group S_4 has exactly three Sylow 2-subgroups.

(f) **T** **F** The automorphism group $\mathrm{Aut}(\mathbb{Z}/18\mathbb{Z})$ has order 6.

Problem 2. (25 points) **A group of order 195.**

Let G be a group of order $|G| = 195 = 3 \cdot 5 \cdot 13$.

(a) (6 points) Determine all values permitted by Sylow's Theorem for n_3, n_5 and n_{13}, and deduce that $n_5 = n_{13} = 1$.

(b) (8 points) Prove that G has a normal subgroup of order 65.

(c) (6 points) Prove that G is solvable by exhibiting a chain of subgroups with abelian quotients.

Problem 3. (25 points) **Invariant factors and elementary divisors.**

(a) (8 points) Let $A = \mathbb{Z}_{20} \times \mathbb{Z}_{30} \times \mathbb{Z}_{50}$. Determine $|A|$ in prime-power form, the elementary divisors of A, and the invariant factors of A.

(b) (4 points) How many abelian groups are there of order $|A|$, up to isomorphism? (Explain briefly; do **not** list them.)

(c) (8 points) Determine, up to isomorphism, all abelian groups of order $392 = 2^3 \cdot 7^2$, in invariant factor form.

Problem 4. (20 points) **A direct product.**

Let $G = A_4 \times \mathbb{Z}/15\mathbb{Z}$.

(a) (4 points) Determine $|G|$ and $Z(G)$.

(b) (4 points) Determine the number of Sylow 3-subgroups of G.

(c) (4 points) Determine the largest order of an element of G.

(d) (3 points) Determine the number of Sylow 2-subgroups of G.