MATH 3551 A — Abstract Algebra I
Practice Midterm 3

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For practice — Midterm 3 is Monday, November 30, 2026 • Time limit: 50 minutes • Total: 100 points

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NAME (please print): ____

Directions. This exam is **closed book**: no textbook, no notes, no calculators, no phones, no other devices, and no assistance of any kind. Write your answers on the exam itself; use the backs of pages if you need more room, and say clearly when you have done so. *Show your work and name the theorems you use* — "by Lagrange's Theorem, $|H|$ divides 24" earns nearly all of the credit, an unjustified correct number earns little. Point values are marked on each problem. Good luck.

About this practice exam. It was drafted with the help of Claude Opus 5, running in a Cowork harness, working from previous exams in this course. **It has not yet been timed or proofread.** It is meant to run well inside a single class period, with time to spare. *Please hunt for mistakes.* If you find a wrong answer, an ambiguous or unanswerable problem, a problem that leans on something we have not covered, or one that takes far longer than its point value suggests, tell me — in class, in office hours, or by email. I would much rather hear it from you now than print it on the real exam.

Problem 1. (30 points) **True or false; prove or disprove.**

Circle **T** or **F**, and then give a one-line reason: a theorem, a short computation, or an explicit counterexample. Three points each.

(a) **T** **F** Every group of order 35 is cyclic.

(b) **T** **F** There is a simple group of order 351.

(c) **T** **F** The alternating group A_4 is simple.

(d) **T** **F** For a prime p there are exactly six abelian groups of order p^5 up to isomorphism.

(e) **T** **F** $\mathbb{Z}/12\mathbb{Z} \times \mathbb{Z}/18\mathbb{Z} \cong \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/36\mathbb{Z}$.

(f) **T** **F** Any two Sylow p-subgroups of a finite group are isomorphic.

(g) **T** **F** The symmetric group S_4 has exactly three Sylow 2-subgroups.

(h) **T** **F** Every group of order 100 is abelian.

(i) **T** **F** A finite abelian group with invariant factors $n_1 \mid n_2 \mid \cdots \mid n_t$ contains an element of order m if and only if $m \mid n_t$.

(j) **T** **F** A_5 has a subgroup isomorphic to $\mathbb{Z}/6\mathbb{Z}$.

Problem 2. (20 points) **A group of order 195.**

Let G be a group of order $|G| = 195 = 3 \cdot 5 \cdot 13$.

(a) (6 points) Determine all values permitted by Sylow's Theorem for n_3, n_5 and n_{13}, and deduce that $n_5 = n_{13} = 1$.

(b) (8 points) Prove that G has a normal subgroup of order 65.

(c) (6 points) Prove that G is solvable by exhibiting a chain of subgroups with abelian quotients.

Problem 3. (20 points) **Invariant factors and elementary divisors.**

(a) (8 points) Let $A = \mathbb{Z}_{20} \times \mathbb{Z}_{30} \times \mathbb{Z}_{50}$. Determine $|A|$ in prime-power form, the elementary divisors of A, and the invariant factors of A.

(b) (4 points) How many abelian groups are there of order $|A|$, up to isomorphism? (Explain briefly; do **not** list them.)

(c) (8 points) Determine, up to isomorphism, all abelian groups of order $392 = 2^3 \cdot 7^2$, in invariant factor form.

Problem 4. (15 points) **A direct product.**

Let $G = A_4 \times \mathbb{Z}/15\mathbb{Z}$.

(a) (4 points) Determine $|G|$ and $Z(G)$.

(b) (4 points) Determine the number of Sylow 3-subgroups of G.

(c) (4 points) Determine the largest order of an element of G.

(d) (3 points) Determine the number of Sylow 2-subgroups of G.

Problem 5. (15 points) **The conjugacy classes of A_5.**

(a) (5 points) List the cycle types of the elements of A_5 and the number of elements of each type.

(b) (5 points) Determine the sizes of the conjugacy classes of A_5. (Careful: a single S_5-class can split into two A_5-classes.)

(c) (5 points) Use (b) to prove that A_5 is simple.

End of practice exam. Total: $30 + 20 + 20 + 15 + 15 = 100$ points.