MATH 3551 A — Abstract Algebra I

Suggested Problems

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How to use this list

Every problem below is an exercise from Dummit and Foote, *Abstract Algebra*, 3rd edition, cited by section and exercise number. The list is organized by section, in the order we cover them.

These problems are not collected and they are not graded. There is no graded homework in this course. Nothing here counts toward your grade directly, and everything here counts toward it indirectly, because the exams are drawn from this material.

The exam problems will be similar but not identical. I am not going to reprint these problems on a midterm. I will ask something of the same kind: the same definition, the same argument shape, a different group or a different number. So the goal is not to have the answers to these particular exercises. The goal is to be able to handle a problem of this kind, cold, in fifty minutes, without the book. Those are different skills, and only the second one is being examined.

Work in groups. Talk about these with each other. Argue about them. Explaining a proof out loud to somebody who is not convinced is the fastest way to find out that you do not actually believe it yourself. There is no academic-integrity concern here — nothing on this list is being submitted — so collaborate freely and often.

Attempt before you look anything up. A problem you have struggled with for fifteen minutes and then seen solved teaches you something. A problem whose solution you read first teaches you almost nothing, and it feels exactly the same at the time, which is what makes it dangerous. Give each problem a real attempt — write something down, get stuck somewhere specific — before you consult the text, a classmate, or me.

Reading the list

For each section you will find:

- **Problems** — four to eight exercises. Start here.
- **Also do** — more of the same, plus the exercises that are interesting rather than typical. Both lists are recommended and neither is ranked.
- **Note** — where an exercise is the source of a theorem we use later, where the difficulty spikes, and where a problem has shown up on a past exam in this course.

Roughly fifteen to twenty-five exercises per section. Two sections (§1.5 and §4.6) have fewer exercises in the book than that, and are handled specially; see the notes there.

What is on which exam

- **Midterm 1**, Monday October 12 — §1.1 through §3.2.
- **Midterm 2**, Friday November 6 — §3.3 through §4.4.
- **Midterm 3**, Monday November 30 — §4.5 through §5.2.
- **Final**, Thursday December 17 — cumulative, and the only exam that reaches §5.4, §5.5 and §6.1.

Note that §5.3 (the table of groups of small order) is not on the syllabus. It is four pages, it is worth reading, and nothing will be asked about it.

Part I. Through Midterm 1 (§1.1–§3.2)

§1.1 Basic Axioms and Examples

Fluency with the group axioms, with orders of elements, and with $\mathbb{Z}/n\mathbb{Z}$ *and* $(\mathbb{Z}/n\mathbb{Z})^\times$ *— the examples every later chapter is built out of.*

Problems. 1, 6, 13, 14, 20, 23, 32

Also do. 9, 10, 16, 19, 21, 22, 24, 26, 31, 33, 36

Note. Exercise 26 is the subgroup criterion; it is used constantly from §2.1 onward, and the text does not prove it, so this exercise is the proof. Exercise 24 is quoted by §2.1 #6. Exercise 31 (a finite group of even order has an element of order 2) is the first real difficulty spike — it is a pairing argument, not a computation. Exercise 36 classifies groups of order 4 and is the first classification theorem you will own outright; it returns at §2.5 #10.

§1.2 Dihedral Groups

D_{2n} *is the standing example of a nonabelian group. Learn to compute with* r*,* s *and the relation* $rs = sr^{-1}$*, not with pictures of polygons.*

Problems. 1, 2, 3, 4, 5, 8

Also do. 6, 7, 9, 10, 13, 14, 15, 16, 17

Note. Exercise 3 (every element not a power of r has order 2, so $D_{2n} = \langle s, sr\rangle$) gets used repeatedly. Exercises 4 and 5 compute $Z(D_{2n})$ and are what §2.2 #7 depends on. Exercises 9–13 (rigid motions of the Platonic solids) are geometry rather than algebra; they come back at §1.7 #20–22, §3.5 #7 and §4.5 #42–43.

§1.3 Symmetric Groups

Cycle notation, and the fact that the order of a permutation is the lcm of its cycle lengths — the computational engine of the whole course.

Problems. 1, 2, 3, 9, 11, 13, 15, 16

Also do. 4, 5, 6, 7, 8, 10, 14, 17, 18, 19, 20

Note. Exercise 15 (order = lcm of cycle lengths) is the source of a fact used on every exam after this one. Prove it once, properly, and you can quote it forever. Exercise 10 is the technical lemma behind both 11 and 15. Exercise 16 (counting m-cycles in S_n) appeared as a true/false item on Dummit's Fall 2014 Exam I.

§1.4 Matrix Groups

$GL_n(F)$ *as a source of finite nonabelian groups, and a first look at the field* $\mathbb{F}_p$*.*

Problems. 1, 2, 3, 4, 7

Also do. 5, 6, 8, 9, 10, 11

Note. This section has only eleven exercises, so all of them are listed. Exercise 7 (the order of $GL_2(\mathbb{F}_p)$) is the counting argument worth doing carefully; do not just quote the formula in the text. Exercise 11 introduces the Heisenberg group, which reappears at §2.2 #14, §5.4 #20 and §5.5 #25. Exercise 10(c) is an application of §1.1 #26.

§1.5 The Quaternion Group

Q_8: the other nonabelian group of order 8, and the standard counterexample to almost everything you might guess.

Problems. 1, 2, 3

Also do. §1.6 #7; §2.5 #6(b), #8(b); §3.1 #32; §3.4 #2; §3.5 #11; §4.2 #4, #7; §4.3 #2(b); §4.4 #4; §5.1 #5, #6

Note. Dummit and Foote give only three exercises in this section, and doing all three takes twenty minutes. Since D_8 versus Q_8 is a recurring exam theme, the additional practice above is drawn from later sections; come back to it as we reach them. §3.5 #11 (S_4 has no subgroup isomorphic to Q_8) was problem 8 on the Fall 2019 final in this course.

§1.6 Homomorphisms and Isomorphisms

What it means for two groups to be the same, and — much more often needed — the standard ways to prove that two groups are not.

Problems. 1, 2, 4, 6, 7, 17, 18, 20

Also do. 3, 5, 8, 9, 11, 13, 14, 15, 21, 22, 24, 26

Note. Exercise 2 (an isomorphism preserves orders of elements) is the workhorse behind every non-isomorphism proof in the course; exercises 4, 5, 6, 7, 8 and 9 are all applications of it, and you should be able to produce any of them in two lines. Exercises 13 and 14 define image and kernel before §3.1 does. Exercise 20 (that $\mathrm{Aut}(G)$ is a group) is the definition §4.4 is built on. Exercise 24 (two distinct elements of order 2 generating a group give D_{2n}) is a difficulty spike.

§1.7 Group Actions

The action axioms and the associated permutation representation — the single most important idea in Chapters 4 and 5, introduced here in its easiest form.

Problems. 1, 2, 3, 4, 5, 6, 13, 16

Also do. 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21

Note. Exercise 4 defines the stabilizer and the kernel of an action; these are exactly §2.2's subject, so do it before §2.2. Exercise 16 (conjugation is an action) is the basis of the class equation in §4.3. Exercise 18 (orbits partition the set) is proved in the text only in §4.1. Exercise 19 is where Lagrange's Theorem first appears in this book — the authors say so in the preface — and we will not prove it until §3.2. Exercise 3 was problem 3 on Dummit's Fall 2014 Exam I, verbatim.

§2.1 Subgroups: Definition and Examples

The subgroup criterion, and the habit of checking it quickly and correctly.

Problems. 1, 2, 6, 8, 10, 13

Also do. 3, 4, 5, 7, 9, 11, 12, 14, 15, 16, 17

Note. Exercise 6 (the torsion subgroup of an abelian group) quotes §1.1 #24 and returns at §3.1 #14. Exercise 8 ($H \cup K$ is a subgroup iff one contains the other) is a perennial true/false item. Exercise 4 is the example that shows why "closed under the operation" is not enough in the infinite case — worth ten minutes even though it looks like a triviality. Exercise 9 defines $SL_n(F)$, which is used at §2.4 #9–11, §3.1 #35 and §4.5 #9–11.

§2.2 Centralizers and Normalizers, Stabilizers and Kernels

The four subgroups attached to a subset. Telling them apart, and computing them, is most of the work in this section.

Problems. 1, 2, 4, 5, 6, 7, 10

Also do. 3, 8, 9, 11, 12, 13, 14

Note. All fourteen exercises are listed; the section is short and every one of them is worth doing. Exercise 5 (for $G = D_8$ and $A = \{1, s, r^2, sr^2\}$, showing $C_G(A) = A$ and $N_G(A) = G$) is the standard exam computation in this section. Exercise 7 computes $Z(D_{2n})$ and leans on §1.2 #4–5. Exercise 6(b) ($H \leq C_G(H)$ iff H is abelian) gets quoted later.

§2.3 Cyclic Groups and Cyclic Subgroups

The complete classification of cyclic groups and of all their subgroups — the one family of groups about which you should be able to answer any question asked.

Problems. 1, 3, 6, 10, 12, 14, 16, 25

Also do. 2, 4, 5, 7, 8, 9, 11, 13, 15, 17, 20, 23, 24, 26

Note. Exercise 16 ($|xy|$ divides $\operatorname{lcm}(|x|, |y|)$ when x and y commute) is used constantly in Chapter 5, and the hypothesis that they commute is not decoration. Exercise 24 is a difficulty spike and previews §4.4. Exercises 21–23 form a number-theoretic sequence ending in "$(\mathbb{Z}/2^n\mathbb{Z})^\times$ is not cyclic for $n \geq 3$"; read them even if you do not do them, since that fact is quotable. Dummit's Fall 2014 Exam I problem 5 is essentially #1 and #3 for a cyclic group of order 270.

§2.4 Subgroups Generated by Subsets of a Group

$\langle A \rangle$ as the smallest subgroup containing A, and recognizing a familiar group from a set of generators and relations.

Problems. 1, 2, 4, 5, 6, 7, 8, 14

Also do. 3, 9, 10, 11, 12, 13, 15, 16, 18, 19, 20

Note. Exercises 6, 7 and 10 are the "identify this group from its generators" drill, which both Midterm 1 and Midterm 2 like. Exercise 16 defines maximal subgroups, needed for the Frattini material in §6.1. Exercise 18 (the group of p-power roots of unity) returns at §3.3 #8, and exercises 19–20 (divisible groups) at §3.1 #15 and §5.2 #16.

§2.5 The Lattice of Subgroups of a Group

Reading a lattice diagram, and using it to settle containment, centralizer and normalizer questions at a glance instead of by computation.

Problems. 2, 3, 6, 7, 8, 9, 10

Also do. 1, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Note. Exercise 10 (with §1.1 #36) classifies groups of order 4. Exercises 11 and 14 introduce the quasidihedral and modular groups of order 16, which reappear at §3.1 #18–19, §3.3 #5–6 and §5.5 #17; if you skip them now you will meet them unprepared later. The lattice of D_{16} on page 70 is the diagram Midterm 1 is most likely to hand you, so exercises 2, 5, 7 and 19 are good value. A warning that is also a true/false item: two groups with identical subgroup lattices need not be isomorphic (Dummit's Fall 2014 Exam I 1(f) and final 1(l)).

§3.1 Quotient Groups and Homomorphisms: Definitions and Examples

Normality, cosets as the elements of a new group, and the first isomorphism theorem used in practice rather than admired in the abstract.

Problems. 3, 8, 9, 14, 22, 24, 25, 36

Also do. 1, 2, 4, 5, 6, 7, 11, 15, 17, 26, 32, 33, 35, 40, 41, 42

Note. Exercise 36 ($G/Z(G)$ cyclic $\Rightarrow$ G abelian) is a genuine theorem that gets quoted on later exams — do it. Exercise 25(a) gives the normality criterion you will actually use ($gNg^{-1} \subseteq N$ for all g), and 25(b) shows why the finiteness hypothesis in exercise 27 is not removable. Exercise 41 defines the commutator subgroup, which is the subject of §5.4 and §6.1. Exercise 14 ($\mathbb{Q}/\mathbb{Z}$) is the standard source of examples of elements of every finite order.

§3.2 More on Cosets and Lagrange's Theorem

Lagrange's theorem, and the arithmetic consequences that fill exam pages.

Problems. 1, 5, 8, 10, 11, 16, 18, 22

Also do. 2, 3, 4, 6, 7, 9, 12, 13, 14, 15, 19, 20, 21, 23

Note. Exercise 9 is a complete proof of Cauchy's Theorem (McKay's argument). It is a difficulty spike, and the payoff is large, because Cauchy is used without comment from §4.5 on. Exercises 16 (Fermat) and 22 (Euler) are the arithmetic core; exercise 23 (the last two digits of $3^{3^{100}}$) is the exam-flavored application. Exercise 17 was problem 2 on Dummit's Fall 2014 final. Exercise 11 (multiplicativity of the index) is used everywhere and is easy to prove and easy to forget.

Part II. Midterm 1 to Midterm 2 (§3.3–§4.4)

§3.3 The Isomorphism Theorems

The second, third and fourth isomorphism theorems, as tools for moving information between G and G/N.

Problems. 1, 2, 3, 4, 7

Also do. 5, 6, 8, 9, 10

Note. There are only ten exercises here; do all of them. Exercise 2 (the Lattice Isomorphism Theorem) is what makes §2.5-style pictures work for quotient groups, and it is the theorem students most often use without noticing. Exercise 9 (P a Sylow p-subgroup, PN/N Sylow in G/N) is the second-isomorphism-theorem route to §4.5 #34. Exercise 3 (a normal subgroup of prime index) is a small, sharp result that shows up in Sylow arguments later.

§3.4 Composition Series and the Hölder Program

Composition factors as the "primes" of finite group theory, and the definition of solvability.

Problems. 1, 2, 4, 5, 7

Also do. 3, 6, 8, 9, 10, 11, 12

Note. Exercise 1 (an abelian simple group is $\mathbb{Z}/p\mathbb{Z}$) is a standard exam item and takes four lines. Exercise 2 (all three composition series of Q_8, all seven of D_8) is the concrete anchor for the whole section — do it before anything else here. Exercises 5 (subgroups and quotients of solvable groups are solvable) and 8 (equivalent characterizations of solvable) are §6.1 material sitting in Chapter 3; if you do not do them now, do them in December. Exercises 6 and 10 are the proofs of Jordan–Hölder itself and are harder than anything that will be asked.

§3.5 Transpositions and the Alternating Group

That the sign of a permutation is well defined, and the group A_n.

Problems. 1, 2, 3, 4, 5, 9, 11

Also do. 6, 7, 8, 10, 12, 13, 14, 15, 16, 17

Note. Exercises 3, 4 and 5 are the generation results. Exercise 5 ($S_p = \langle \sigma, \tau \rangle$ for any transposition σ and any p-cycle τ) appeared on the Fall 2019 final in this course in the sloppier form "S_n is generated by any 2-cycle and any n-cycle" — which is false, and the primality is exactly what makes exercise 5 true. That is the kind of distinction a true/false question is testing. Exercise 11 (S_4 has no subgroup isomorphic to Q_8) was problem 8 on that same final. Exercise 10 (a composition series for A_4) links back to §3.4.

§4.1 Group Actions and Permutation Representations

Orbits, stabilizers, and the orbit–stabilizer relationship in the form you will actually use it.

Problems. 1, 2, 3, 4, 5

Also do. 6, 7, 8, 9, 10

Note. Only ten exercises; do all of them. Exercise 1 (stabilizers of points in a common orbit are conjugate) is the source of the conjugacy statements in §4.3 and §4.5, and it is one line once you see it. Exercise 4

(S_3 acting on ordered pairs, relabelled as a subgroup of S_9) is the model for the "action on a labelled set" problem that appears on essentially every exam in this course; Dummit's Fall 2014 Exam II problem 2 is the D_8-on-six-pairs version of it. Do exercise 4 and then invent two more like it.

§4.2 Cayley's Theorem

Every group is a permutation group; and, more useful, a subgroup of index n produces a homomorphism into S_n.

Problems. 1, 2, 3, 7, 8, 9, 13

Also do. 4, 5, 6, 10, 11, 12, 14

Note. All fourteen are listed. Exercise 8 (a subgroup of index n contains a normal subgroup of index dividing $n!$) is the tool behind a large fraction of the "prove G is not simple" arguments in §4.5; learn it here, where it is easy, rather than there, where it is buried. Exercises 11–13 form a sequence ending in "if $|G| = 2k$ with k odd then G has a subgroup of index 2," which is a satisfying and genuinely clever result.

§4.3 The Class Equation

Conjugacy classes, the class equation, and conjugacy in S_n by cycle type.

Problems. 2, 5, 7, 8, 11, 13, 33

Also do. 3, 4, 6, 9, 10, 12, 19, 20, 22, 23, 24, 25, 28, 29, 30

Note. Exercise 33 gives the formula for the size of a conjugacy class in S_n; it is behind Dummit's Fall 2014 Exam II 1(h) and problem 7(c) of the Fall 2019 final, both of which ask whether the class of $(1\,3)(5\,7)$ in S_7 has 210 elements. If you can compute that number in under a minute you are ready for this part of the exam. Exercise 29 (p-groups have a subgroup of order p^β for every $\beta \leq \alpha$) is the class-equation payoff and is used in §4.5 and §6.1. Exercise 24 (G is not the union of the conjugates of a proper subgroup) is a difficulty spike and is worth attempting anyway.

§4.4 Automorphisms

$\mathrm{Aut}(G)$*, inner automorphisms, characteristic subgroups, and the fact that* $\mathrm{Aut}(\mathbb{Z}/n\mathbb{Z}) \cong (\mathbb{Z}/n\mathbb{Z})^\times$*.*

Problems. 1, 4, 6, 7, 8, 13, 15, 17

Also do. 2, 3, 5, 9, 10, 11, 12, 14, 16, 18, 20

Note. Exercise 1 ($\mathrm{Inn}(G) \trianglelefteq \mathrm{Aut}(G)$) and exercises 6–8 (characteristic subgroups) are the definitions that §5.4 and §6.1 depend on; exercise 6's requested counterexample (a normal subgroup that is not characteristic) is the one people cannot produce under pressure. Exercises 12, 13 and 14 are a family of "a normal subgroup of suitably coprime order is central" arguments and one of them is a very likely Midterm 2 problem — do all three, they are the same argument. Exercises 15, 16 and 17 give you the concrete automorphism groups; Dummit's Fall 2014 final 1(c) asks for $|\mathrm{Aut}(\mathbb{Z}/72\mathbb{Z})|$ and 1(j) for $(\mathbb{Z}/24\mathbb{Z})^\times$. Exercise 18 (every automorphism of S_n is inner for $n \neq 6$) is long, hard and beautiful; read it, attempt it if you have the time, and do not expect it on an exam.

Part III. Midterm 2 to Midterm 3 (§4.5–§5.2)

§4.5 The Sylow Theorems

Existence, conjugacy and counting of Sylow subgroups, and — what actually matters — the handful of argument shapes that use them.

Problems. 1, 13, 14, 15, 17, 19, 20, 33

Also do. 2, 3, 4, 5, 6, 7, 8, 12, 16, 18, 21, 22, 23, 24, 26, 29, 34, 36

Note. This is the largest and most exam-relevant section in the course; the book has fifty-six exercises and you should do more of them than you want to. Exercise 13 (order 56) is the model counting argument, and 14 (312), 15 (351), 19 (6545) and 20 (1365) are the same argument with the numbers changed — which is precisely what an exam problem is. Exercise 16 ($|G| = pqr$) and exercise 29 are the structural statements. Exercise 34 ($P \cap N$ is a Sylow p-subgroup of N) should be done with the conjugacy part of Sylow, not with the second isomorphism theorem; compare §3.3 #9 and notice that the two routes teach different things. Dummit's Fall 2014 Exam II problem 6 ($|G| = 45$) and final problem 5 ($|G| = 520$) are exactly this genre. Work exercises 17–28 until the shape is automatic.

§4.6 The Simplicity of A_n

A_n is simple for $n \geq 5$: the first nonabelian simple groups, and the reason the Hölder program is hard.

Problems. 1, 2, 4

Also do. 3, 5, 6, 7, 8

Note. The book gives only eight exercises here, all of them listed. Exercise 4 (A_n is generated by 3-cycles) is the one to know cold: it is used inside the simplicity proof, and Dummit's Fall 2014 final 1(b) is the $n = 5$ case. Exercise 2 (the normal subgroups of S_n for $n \geq 5$) follows from simplicity in three lines and makes a clean exam problem. For more problems of this flavor see §3.5 #16–17 and §4.5 #29, #41–43, the last of which realize A_5 as $SL_2(\mathbb{F}_4)$ and as the rotation group of the icosahedron.

§5.1 Direct Products

How the order, center, subgroups and Sylow subgroups of a direct product are assembled from those of the factors.

Problems. 1, 4, 5, 7, 10, 11

Also do. 2, 3, 6, 8, 12, 14, 15, 16, 17, 18

Note. Exercise 1 (the center of a product is the product of the centers) and exercise 4 (Sylow subgroups of a product) are the two facts you will end up using without thinking about them, so prove them once. Exercise 5 (a nonnormal subgroup of $Q_8 \times \mathbb{Z}_4$, even though every subgroup of each factor is normal) is the counterexample that stops you believing something false, and is exactly the sort of thing a true/false question is built from. Exercise 18 is a list of "give an example of a group with property X" — it is the best true/false preparation on this list.

§5.2 The Fundamental Theorem of Finitely Generated Abelian Groups

Invariant factors, elementary divisors, and translating between the two.

Problems. 1, 2, 3, 4, 5, 9

Also do. 6, 7, 8, 11, 12, 13, 15, 16

Note. Exercises 1–4 are the mechanical drill, and you should expect one of them on Midterm 3 and another on the final. Dummit's Fall 2014 Exam II problem 4 is exercises 1 and 4 for order $648 = 2^3 3^4$. Exercise 5 (a finite abelian group of type $(n_1, \ldots, n_t)$ contains an element of order m exactly when m divides n_t) is the fact that makes the true/false questions about abelian groups answerable at speed. This section is short and entirely learnable; there is no excuse for losing points here.

Part IV. After Midterm 3 — final exam only (§5.4–§6.1)

Everything in this part is examined only on the final. It is also, in my experience, the material students find hardest, and the recess sits in the middle of it. Start early.

§5.4 Recognizing Direct Products

Commutators, the commutator subgroup, and the criterion that lets you recognize G as $H \times K$ from the inside.

Problems. 1, 2, 4, 10, 15, 16

Also do. 3, 5, 7, 8, 11, 12, 13, 17, 18, 19, 20

Note. Exercise 10 (a finite abelian group is the direct product of its Sylow subgroups) is the structural fact that ties §4.5 to §5.2, and it is a good final-exam problem. Exercises 2 ($H \trianglelefteq G$ iff $[G,H] \leq H$) and 16 ($K \trianglelefteq G \Rightarrow K' \trianglelefteq G$) are the commutator facts §6.1 needs. Exercise 4 (the commutator subgroups of S_4 and A_4) is the concrete anchor; exercise 5 ($A_n = S_n'$ for $n \geq 5$) connects to §4.6. Exercise 19 (perfect groups) is enrichment.

§5.5 Semidirect Products

Building a group out of H, K and an action of K on H; and using it to classify groups of small order.

Problems. 1, 2, 3, 8, 11, 12

Also do. 4, 5, 6, 10, 13, 16, 17, 20, 21, 24

Note. The difficulty genuinely spikes here; this section is harder than anything before it, and the reason is that you are now constructing groups rather than analyzing given ones. Exercises 11 (order 28), 12 (order 20) and 8 (order 75) are the classification problems to do, in that order. Dummit's Fall 2014 final problem 4 (the four groups of order $70 = 2 \cdot 5 \cdot 7$) and 1(n) (order 75) are exactly these. Exercise 24 (every group of order n is abelian iff n is squarefree and $\ldots$) is the capstone. Exercises 7 (thirteen groups of order 56) and 14 (thirteen groups of order 60) are multi-day projects, not exam problems — read them for the method.

§6.1 p-groups, Nilpotent Groups, and Solvable Groups

Nilpotent and solvable groups, the upper and lower central series, and the derived series.

Problems. 3, 6, 7, 9, 10, 12

Also do. 1, 2, 4, 8, 14, 15, 17, 19, 23, 26

Note. Only §6.1 is on the syllabus, and within it only the solvability and nilpotence material; the Frattini subgroup exercises (21–30) are optional enrichment and will not be examined. Exercises 9 (G is nilpotent iff elements of coprime order commute) and 3 (a normal subgroup of each order dividing $|G|$) are the two characterizations to know. Exercise 10 (D_{2n} is nilpotent iff n is a power of 2) is the concrete test case, and exercise 12 (upper and lower central series for A_4 and S_4) is the computation most likely to be asked. If you have not already done §3.4 #5 and #8, do them now: "every abelian group is solvable" was a true/false item on the Fall 2019 final in this course, and the chain of implications behind it lives partly in §3.4 and partly here.

A closing note on the exams

Three things are worth saying plainly about how these problems relate to what you will be asked.

True/false questions are not filler. Past exams in this course have opened with ten to twenty true/false items worth thirty to sixty points — a third of the exam or more. They are where a shallow acquaintance with the material is detected. Almost every one of them is a counterexample you either have or do not have: is $S_4 \cong D_{24}$, does A_4 have a normal subgroup of order 4, is there an element of order 21 in S_{10}, does a subgroup lattice determine a group. The "give an example of a group such that…" exercises (§5.1 #18 above all, and scattered through §2.1, §3.1 and §4.4) are the best possible preparation for them, and they are the exercises students most often skip.

Computation and proof are both examined. Roughly half of a typical exam is computation you must get exactly right — a cycle decomposition, an order, a count of subgroups, a list of invariant factors — and roughly half is a proof of three to ten lines. Practising only the proofs leaves you slow and error-prone on the first half; practising only the computations leaves you with nothing to write on the second.

Say which theorem you are using. On an in-class exam I would much rather see "by Lagrange, $|H|$ divides 24, so $|H| \in \{1, 2, 3, 4, 6, 8, 12, 24\}$" than a correct final answer with no justification. Naming the theorem is most of the credit, and it is a habit you build by writing out solutions to the problems above rather than by convincing yourself in your head that you could.