

Math 163 — Test 01

Monday September 17th 2012

**Instructions** Remember to show all your work so you can get partial credit. You shouldn't need a calculator on this test. Please leave answers in their exact form. Try not to overthink the problems too much.

1. (15 Points) Find the derivatives of the following functions

(a)  $f(x) = \ln(x)$ .

(b)  $h(t) = e^{t+1}$ .

(c)  $g(x) = 2^x$

2. (15 Points) Find the following integrals

(a)  $\int_1^x \frac{1}{t} dt$

(b)  $\int \frac{1}{1+4x^2} dx$

(c)  $\int \tan(\theta) d\theta$

3. (10 Points) Graph the following functions. Make sure to label key features.

(a)  $f(x) = \ln(x)$ .

(b)  $f(x) = 1 + e^{x-2}$ .

4. (10 Points) Find the following limits using L'hôpital's rule

(a)  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$

(b)  $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$

5. (15 points) Find the following limits. (You can just state the answer if you know it.)

(a)  $\lim_{x \rightarrow \infty} e^{-x} \cos(x)$

(b)  $\lim_{x \rightarrow \infty} \tan^{-1}(x)$

(c)  $\lim_{x \rightarrow \infty} \frac{2e^x - e^{-x}}{e^x + e^{-x}}$

6. (15 Points) Find an inverse function for the following functions and state their domains.

(a)  $f(x) = e^x$

(b)  $h(x) = e^{2x} + 1$

(c)  $g(x) = x^2 + 4x + 4$  when  $x \leq -2$ . (It might be helpful to graph this function)

7. (15 Points) Let  $f^{-1}(t)$  be the inverse function of  $f(s) = s + e^s$ . Find the line tangent to the graph of  $f^{-1}(t)$  at  $t = 1$ . (Hint: don't try to compute the inverse directly like in problem 6 )

8. (15 Points) Derive the formula for the derivative of  $\sin^{-1}(x)$  where the domain of  $\sin(x)$  is taken to be  $[-\pi/2, \pi/2]$ .