Math 163 — Test 03

Monday, November 26th 2012

Instructions Remember to show all your work so you can get partial credit. Please leave answers in their exact form. You won't need a calculator on this test but you can use one if it doesn't compute Taylor Series.

1. (15 Points) Do the sequences converge or diverge as $n \to \infty$? If they converge please state the limit.

   (a) $a_n = 1 + 1/n$

   (b) $b_n = (-1)^n$

   (c) $c_n = 1 + (-1)^n/n$

2. (15 Points) Do the series converge or diverge? If they converge please find the limit. If they diverge explain why.

   (a) $\sum_{n=1}^{\infty}(1-1/n)$

   (b) $\sum_{n=1}^{\infty}\frac{\ln(n)}{\sqrt{n}}$

   (c) $\sum_{j=1}^{\infty}2^{-j}$

3. (10 Points) Let $f(x) = x^3 + 1$.

   (a) Find the Taylor Series for $f(x)$ centered at $x_0 = 0$.

   (b) Find the Taylor Series for $f(x)$ centered at $x_0 = -1$.

4. (20 Points) Find the Taylor Series expansion of $g(x) = \frac{1}{1+4x^2}$ at $a = 0$ and compute its radius of convergence.

5. (20 Points) Find the Taylor Series expansion of $f(x) = e^{2x} - 1$ at $a = 1$ and compute its radius of convergence.

6. (20 Points) For parts (a), (b) and (c), if you know the formula for the particular Taylor Series and the radius of convergence you can just state your answer. If you are unsure and want partial credit give a justification.

   (a) Find the Taylor Series for e^t around $t_0 = 0$. What is the radius of convergence?

   (b) Find the Taylor Series for $\cos(t)$ around $t_0 = 0$. What is the radius of convergence?

   (c) Find the Taylor Series for $\sin(t)$ around $t_0 = 0$. What is the radius of convergence?

   (d) Prove Euler's Formula:

$$e^{it} = \cos(t) + i\sin(t),$$

   where $i = \sqrt{-1}$.