1

(a) $\lim_{n\to\infty} 1 + 1/n = 1$, converges

(b) $\lim_{n\to\infty} (-1)^n$ DNE, diverges

(c) $c_n = \lim_{n\to\infty} 1 + \frac{(-1)^n}{n} = 1$, converges

2

(a) $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ by the p-test with $p=1$

(b) $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}} \geq \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ which diverges by the p-test with $p = 1/2$

$\Rightarrow \sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$ diverges

(c) $\sum_{j=1}^{\infty} 2^{-j} = \sum_{j=0}^{\infty} \left(\frac{1}{2}\right)^j - 1$

$= \frac{1}{1 - 1/2} - 1$

$= 2 - 1$

$= 1$

(3) Let $f(x) = x^3 + 1$

(a) x^3+1 is the Taylor series centered at $x_0 = 0$

(b) $x^3 + 1 = (x+1-1)^3 + 1$

$= \left[(x+1)^3 - 3(x+1)^2 + 3(x+1) - 1\right] + 1$

$= 3(x+1) - 3(x+1)^2 + (x+1)^3$

power series centered at $x_0 = -1$.

(4) $g(x) = \dfrac{1}{1+4x^2} = \dfrac{1}{1+(2x)^2}$

$= \dfrac{1}{1-(-(2x)^2)}$

$= \sum_{n=0}^{\infty} (-(2x)^2)^n = \sum_{n=0}^{\infty} (-1)^n 4^n x^{2n}$

Radius of convergence:

$|-(2x)^2| < 1 \Rightarrow |4x^2| < 1$

$\Rightarrow |x| < 1/2$

radius of convergence

5

$$f(x) = e^{2x} - 1,$$

$$f'(x) = 2e^{2x} \qquad f(0) = 0$$

$$f''(x) = 4e^{2x}$$

$$f'''(x) = 8e^{2x} \qquad f^{(n)}(1) = 2^n e^2, \quad n \geq 1$$

$$f^{(n)}(x) = 2^n e^{2x}, \quad n \geq 1$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!}(x-1)^n$$

$$= \sum_{n=1}^{\infty} \frac{2^n e^2}{n!}(x-1)^n$$

To Find the radius and interval of convergence we apply the ratio test,

$$\frac{|a_n|}{|a_{n+1}|} = \left(\frac{2^n e^2}{n!}\right) \Big/ \frac{2^{n+1} e^2}{(n+1)!}$$

$$= \frac{(n+1)!}{2^{n+1} \cancel{e^2}} \cdot \frac{2^n \cancel{e^2}}{n!} = \frac{n+1}{2} \to \infty$$

as $n \to \infty$

$\Rightarrow$ there is an infinite radius of convergence.

(a) $$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!},$$ infinite radius of convergence

(b) $$\cos(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!},$$ infinite radius of convergence

(c) $$\sin(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!},$$ infinite radius of convergence

(d) $$e^{it} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!}$$

$$= \underbrace{\sum_{n=0}^{\infty} \frac{(it)^{2n}}{(2n)!}}_{\text{even terms}} + \underbrace{\sum_{n=0}^{\infty} \frac{(it)^{2n+1}}{(2n+1)!}}_{\text{odd terms}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} + \sum_{n=0}^{\infty} i\frac{(-1)^n t^{2n+1}}{(2n+1)!}$$

$$= \cos(t) + i\sin(t).//$$