Math 163: FALL 2012 HOMEWORK

Below is a list of online problems (go through webassign), and a second set that you need to write up and turn in on the given due date, in class. Each day, you need to work through the online problems first (these are more basic), and then work on the written problems. You should do these problem without using a calculator, unless specified, as you will be expected to do so on the exams.

HOMEWORK DAY 1 — Due Thursday, August 23rd

Online: §6.1: 1,3,7,9,15,20,21,24,29,31,JIT

1. Use the Inverse Function Property (Equations 4, p 386) to check wether f and g are inverses of each other.
$$f(x) = \frac{6-x}{7} \ , \qquad g(x) = -7x + 6$$
2. Consider the function $L(v) = \sqrt{1 - v^2/c^2}$ where c is a positive constant, and $v \geq 0$.
   (a) Sketch the graph of L. (Hint: first factor $1/c^2$ out of the root, then note that the graph is a circle stretched in the y- direction.)
   (b) Is L invertible? If so, find L^{-1}.
   (c) In your plot in (a) showing the graph of L, add the graph of L^{-1}. The graph should clearly show all ranges and domains.
3. (a) Suppose f is differentiable and invertible for all $x \in D$, with $f'(x) \neq 0$. Derive the formula in Theorem 7, page 388, as follows. Start with the relation
$$f(f^{-1}(x)) = x$$
   and differentiate both sides with respect to x. Solve for $(f^{-1})'(x)$. What is the geometric meaning of this formula?
   (b) Suppose the function f is differentiable for all $x \in D$ with $f'(x) \neq 0$. Derive the formula from Theorem 7, page 388 as follows. Start with the relation $f(f^{-1}(x)) = x$ and differentiate both sides with respect to x. For for $(f^{-1})'(x)$. What is the geometric meaning of this formula?
4. Answer the following by inspection (that is, without finding an explicit formula for f^{-1} first).
   (a) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$, $f(f^{-1}(2))$, $(f^{-1})'(3)$.
   (b) If $f(x) = x^3 + 3\sin(x) + 2\cos(x)$, find $(f^{-1})'(2)$.
   (c) If $f(x) = \int_3^x \sqrt{1+t^3}\,dt$, find $f^{-1}(0)$, $(f^{-1})'(0)$, equation for tangent line to f^{-1} at $x = 0$.

HOMEWORK DAY 2 — Due Thursday, August 23rd

Online: §6.2a: 1,2,5,7,9,13,15,16,17,20,22.

1. §6.2, # 8,10,12 (graph exponentials)
2.
   (a) $\displaystyle\lim_{x\to\infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$ (b) $\lim_{x\to-\infty} \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$
   (c) $\displaystyle\lim_{x\to\infty} e^{-2x}\cos(x)$ (d) $\displaystyle\lim_{x\to\infty} (1.0001)^x$

(e) $\displaystyle\lim_{x\to(\pi/2)^+} e^{\tan(x))}$ (f) $\displaystyle\lim_{x\to-\infty}(1.0001)^x$

(g) $\displaystyle\lim_{x\to 0^+} 2^{1/x}$ (h) $\displaystyle\lim_{x\to 0^-} 2^{1/x}$

HOMEWORK DAY 3 — Due Thursday, August 30th

Online: §6.2b: 31,35,37,39,52,57,83,84,85,86,89.

1. §6.2, # 32,34,36 (Find derivatives)

2. §6.2, # 81,87,88 (Find integrals)

3. A drug response curve describes the level of medication in the bloodstream after a drug is administered. A surge function

$$S(t) = At^p e^{-kt}\ , \quad t \geq 0$$

is often used to model the response curve, reflecting an initial surge in the drug level and then a more gradual decline. The parameters A, p, k are all positive and depend on the particular drug.

(a) Find the intervals where S is increasing, and where it is decreasing.

(b) Use your result in (a) to specify the time and the value of maximal response

---

HOMEWORK DAY 4 — Due Thursday, August 30th

Online: §6.3: 3,4,7,8,16,17,18,27,29,39,45,46,47.

1. Sketch the graphs of the following functions. (a) $f(x) = \ln(x)$
   (b) $f(x) = \ln(1/x)$ (c) $f(x) = \ln(x-1)$ (d) $f(x) = \ln|x|$ (e) $f(x) = \ln(x) + 4$

2. §6.3 # 49,50,52 (evaluate limits)

3. §6.3, # 43

4. §6.3, # 65

HOMEWORK DAY 5 — Due Thursday, August 30th

Online: §6.4: 2,3,6,7,8,11,19,20,21,71,73,74,76,77,78,80.

1. §6.4: # 46, 50 (logarithmic differentiation)

2. §6.4: 61 (find intervals of concavity and inflection points)

3. §6.4: 62 (find absolute minimum, make sure your answer is justified)

HOMEWORK DAY 6 — Due Thursday, September 6th

Online: §6.6: 1,3,5,23,25,33,45.

1. *Graphs of inverse trig functions.*
   (a) Sketch the graph of $y = \sin x, -\pi/2 \leq x \leq \pi/2$ and $y = \sin^{-1} x$ on the same screen.
   (b) Sketch the graph of $y = \tan x, -\pi/2 < x < \pi/2$ and $y = \tan^{-1} x$ on the same screen.
   (c) Sketch the graph of $y = 3\tan^{-1}(x) + 2$

2. §6.6 # 11 (find cos(asin(x)))

3. §6.6 #27,32 (derivatives)
4. §6.6, # 38 (implicit)
5. §6.6, # 43,48 (limits)

HOMEWORK DAY 7 — Due Thursday, September 6th

Online: §6.6: 62,64,67,69. §6.7: 1,3,9,31,32,33.

1. §6.6, # 61,65,66,70 (integrals)
2. The hyperbolic tangent is defined as $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$. Use this definition to answer the questions below.
   (a) Is $\tanh(x)$ odd or even? Show your work. (Use the definition of an odd function.)
   (b) Find the limits $\lim_{x\to\infty} \tanh(x)$ and $\lim_{x\to-\infty} \tanh(x)$
   (c) Find all intercepts and all asymptotes of $y = \tanh(x)$.
   (d) Show that $f(x) = \tanh(x)$ is always increasing.
   (e) Use the information above to sketch a graph of $y = \tanh(x)$.
   (f) Show that $\frac{d}{dx}[\tanh(x)] = \text{sech}^2(x)$, where $\text{sech}(x) = \dfrac{2}{e^x + e^{-x}}$.
3. §6.7, #49 (water waves)

HOMEWORK DAY 8 — Due Thursday, September 13th

No Online problems.

1. Find the following limits, if they exist. You must show all work. Use L'Hôpital's rule if applicable (only 10 of these problems require L'Hôpital's). If necessary, use L'Hôpital's rule repeatedly. If necessary, use algebraic manipulation to put the limit in a form that can be treated using L'Hôpital's rule. Remember that if the variable appears in both the base and the exponent, you can begin by taking the limit of the logarithm to "bring the variable down from the exponent".

   (a) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x^2-x}$
   (b) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$
   (c) $\displaystyle\lim_{x\to \pi} \frac{\sin x}{x}$
   (d) $\displaystyle\lim_{x\to \infty} \frac{\sin x}{x}$
   (e) $\displaystyle\lim_{x\to \infty} \frac{\ln(x)}{\sqrt{x}}$
   (f) $\displaystyle\lim_{x\to 0} \frac{x+1-e^x}{x^2}$
   (g) $\displaystyle\lim_{x\to \infty} \frac{x^3}{e^{2x}}$
   (h) $\displaystyle\lim_{x\to \infty} x\ln(x)$
   (i) $\displaystyle\lim_{x\to 0^+} x\ln(x)$
   (j) $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{x}$
   (k) $\displaystyle\lim_{x\to 0^+} \frac{\ln(x)}{x}$
   (l) $\displaystyle\lim_{x\to \infty} \Big(\ln(x) - \ln(x+1)\Big)$
   (m) $\displaystyle\lim_{x\to \infty} xe^{-2x}$
   (n) $\displaystyle\lim_{x\to -\infty} xe^{-2x}$
   (o) $\displaystyle\lim_{x\to \infty} \left(\sqrt{x^2+6} - x\right)$
   (p) $\displaystyle\lim_{x\to -\infty} \left(\sqrt{x^2+6} - x\right)$

(q) $\lim_{n\to\infty}(1+\frac{r}{n})^n$, $r \in \Re$ (r) $\lim_{x\to 0^+} x^{\sqrt{x}}$

2. Which one grows faster? Or do they grow at the same rate? Show your work.
   (a) $\sqrt{x}$ or x^2 ? (b) $\ln x$ or $\log_{10} x$?
   (c) 2^x or e^x ? (d) x^{100} or 1.0001^x ?
3. Order the following functions from slowest growing to fastest growing, as $x \to \infty$. Group functions that grow at the same rate together.
   (a) e^x (b) x^x (c) $\ln(x)^x$
   (d) $e^{x/2}$ (e) 2^x (f) $(0.9)^x$

HOMEWORK DAY 9 — Due Thursday, September 13th

Online: §7.1: 3,5,9,10,12,13,24,25,26,29.

1. Find the following definite and indefinite integrals.
   (a) $\int x\sin(3x)\,dx$ (b) $\int x\sin(3x^2)\,dx$ (c) $\int \frac{dt}{5-3t}$
   (d) $\int \frac{\ln(x)}{x}\,dx$ (e) $\int \sin^5(2x)\cos(2x)\ dx.$ (f) $\int_0^{\pi/2} \cos^5(t)\,dt$
   (g) $\int_0^1 re^{r/2}\,dr$ (h) $\int x^2\ln(3x)\,dx$ (i) $\int_1^2 \frac{\ln(t)}{\sqrt{t}}\,dt$
   (j) $\int x^2\cos(mx)\,dx$ (k) $\int \tan^{-1}(4t)\,dt$ (l) $\int 2^s\,ds$
   (m) $\int_0^t e^s\sin(t-s)\,ds$ (n) $\int \cos^2(4x)\,dx$ (o) $\int \sin(3x)\cos(5x)dx$

HOMEWORK DAY 10 — Due Thursday, September 13th

Online: §7.2: 1,3,7,21,23

1. (a) Evaluate $\int \sin(3x)\cos(5x)\,dx$ using the appropriate identities in §7.2, p500.
   (b) Evaluate $\int_0^{2\pi} \sin(3x)\cos(5x)\,dx$.

HOMEWORK DAY 13 — Due Thursday, September 20th

Online: §7.4a: 12,13,15,19.

§7.4: 16

HOMEWORK DAY 14 — Due Thursday, September 27th

Online: §7.4b: 21,22,23.

§7.4: 26

HOMEWORK DAY 15 — Due Thursday, September 27th

Online: §7.7: 2,3,11

1. §7.7, #1(b,c,d) (answer these without doing part a!)

3. §7.7, #30

HOMEWORK DAY 16 — Due Thursday, September 27th

Online: §7.7: 19

1. §7.7: 20

2. How large should n be to guarantee that the error in the numerical approximation of

$$\int_0^1 e^{x^2}\,dx$$

is less than 10^{-5} if you use
(a) the Trapezoid rule?
(b) Simpson's rule?
Make sure to clearly explain your answers.

HOMEWORK DAY 17 — Due Thursday, October 4th

Online: §7.8: 1,7,9,13,15,21,22,23,25,26

1. §7.8, #14, 16,18,24

HOMEWORK DAY 18 — Due Thursday, October 4th

Online: §7.8: 27,28,32,33.

2. §7.8, # 31, 34, 40

HOMEWORK DAY 19 — Due Thursday, October 4th

Online: §9.1: 3,5,9,11.

1. §9.1, # 4, 7, 10

HOMEWORK DAY 20 — Due Tuesday, October 9th

Online: §9.3: 11,13,10,16.

§9.3: 1, 9, 18

HOMEWORK DAY 21 — Due Tuesday, October 9th

Online: §9.2a: 1,3,4,7,9.

1. The differential equation below models the temperature $T(t)$ of a cup of coffee in a 21C room, t minutes after some initial time. Here, the temperature of the cup of coffee is given in C.

$$\frac{dT}{dt} = -\frac{1}{50}(T - 21)$$

(a) Draw the direction field for the differential equation.
(a) For which initial coffee cup temperature does the temperature remain constant? State the steady solution.
(a) Find the temperature $T(t)$ of the coffee if its initial temperature at $t = 0$ is 85^o.

2. Consider the differential equation $\frac{dx}{dt} = x^2$, where $x = x(t)$.
   (a) Sketch the direction field, as well as a few sample integral curves.
   (b) Find the general solution.
   (c) Find the particular solution that satisfies $x(1) = 2$. Draw this solution in your direction field in (a).
   (d) Find the particular solution that satisfies $x(1) = 0$. Draw this solution in your direction field in (a). (Note: can you use separation of variables in this case? The answer is NO, since the first step would involve dividing by x, and you cannot divide by zero. So how do you find the solution? Answer: either guess and check, or look at direction field.)

HOMEWORK DAY 22 — Due Thursday, October 18th

Online: §9.1: 10, §9.2: 19

1. §9.2, 20 (Euler's method)

2. Consider the initial value problem

$$\frac{dP}{dt} = P(2 - P) \ , \quad P(0) = 1/2$$

   (a) Plot the direction field for this differential equation. Include several solution curves. Clearly indicate the one solution that solves the initial value problem.
   (b) Find the solution $P(t)$ of the initial value problem.
   (c) Find the limits $\lim_{t\to\infty} P(t)$ and $\lim_{t\to-\infty} P(t)$. Do these limits agree with the graph of $P(t)$ that you plotted in (a)?
   (d) Approximate $P(1)$ using Eulers method with $\Delta t = 1/2$. Compare the approximation with the your exact result in (b).

HOMEWORK DAY 23 — Due Thursday, October 18th

Online: §9.4: 1, Example

1. A glucose solution is administered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Thus a model for the concentration $C = C(t)$ of the glucose solution in the bloodstream is

$$\frac{dC}{dt} = r - kC$$

   where r and k are positive constants. Assume that C is measured in mg/mL and t in minutes.
   (a) Sketch the direction field and a few solution curves.
   (b) Assume k is known. At what rate should the solution be administered so that the glucose concentration approaches a desired level of .9 mg/mL?
   (c) Find the solution to the differential equation if $C(0) = r/k$.
   (d) Find the solution to the differential equation if $C(0) = C_0 > r/k$. (Since the initial concentration is bigger than r/k you may assume that $C(t) > r/k$ at all times, in view of your direction field.)

HOMEWORK DAY 26 — Online only

Online: §11.1: 1,5,9,13,14,23,24,25,26,27,28,29,35,38,39,41,43.

HOMEWORK DAY 27 — Due Thursday, October 25th

Online: §11.2a: 1,3,5,17,18,21.

1. (a) What does it mean to state that $\sum_{k=1}^{\infty} a_k = L$?
   (b) Assume $\sum_{k=1}^{\infty} a_k = L$. What is $\lim_{k\to\infty} a_k$? What is $\lim_{n\to\infty} s_n$, where $s_n = \sum_{k=1}^{n} a_k$?

2. Fill in the blanks using either *may* or *must*.
   (a) A series with summands tending to 0 ___________ converge.
   (b) A series that converges ___________ have summands that tend to zero.
   (c) If a series diverges, then the summands ___________ not tend to 0.
   (d) If a series diverges, then the Divergence Test ________ succeed in proving the divergence.
   (e) If $\sum_{n=10}^{\infty} a_n$ diverges, then $\sum_{n=1000}^{\infty} a_n$ ___________ diverge.

3. For the series $\displaystyle\sum_{n=1}^{\infty} 1$, find a formula for the partial sums s_N, the limits $\displaystyle\lim_{n\to\infty} a_n$ and $\displaystyle\lim_{N\to\infty} s_N$, and determine whether the series converges or diverges.

4. Evaluate the following series or determine that they diverge (with very brief explanation).
   (a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$ (b) $\displaystyle\sum_{n=-3}^{\infty} \frac{1}{(-5)^n}$ (c) $1 + 0.4 + 0.16 + 0.064 + \ldots$
   (d) $\displaystyle\sum_{j=1}^{\infty} \cos(\pi j)$ (e) $\displaystyle\sum_{l=1}^{\infty} \sin(2\pi l)$ (f) $\displaystyle\sum_{n=4}^{\infty} 5^{-n}3^{-n}4^{n}$

HOMEWORK DAY 28 — Due Thursday, October 25th

Online: §11.2b: 29,35,38,39,40,43,44.

1. Evaluate the following series or determine that they diverge (with very brief explanation).
   (a) $\displaystyle\sum_{n=3}^{\infty} (5^{-n} + 2 \cdot 3^{-n})$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{7^{n-1}}{(-9)^n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (d) $\displaystyle\sum_{n=1}^{\infty} (\tan^{-1}(n+2) - \tan^{-1} n)$

HOMEWORK DAY 29 — Due Thursday, November 1st

Online: §11.3: 3,4,11,12.

1. Determine whether the following series converge or diverge (with a brief, but complete, explanation).

(a) $\displaystyle\sum_{j=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ (b) $\displaystyle\sum_{j=-2}^{\infty} \frac{2j^2}{3j^2+1}$ (c) $\displaystyle\sum_{j=1}^{\infty} \frac{1}{n^2+1}$

(d) $\displaystyle\sum_{j=1}^{\infty} \frac{1}{n^5}$ (e) $1 + \frac{10}{4} + \frac{10}{9} + \frac{10}{16} \ldots$ (f) $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$

(g) $\displaystyle\sum_{n=100}^{\infty} \frac{4^n}{3^n+3}$ (h) $\displaystyle\sum_{n=2}^{\infty} 2^n$ (i) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$

2. (a) Write the number $0.\overline{2} = 0.22222\ldots$ as a geometric series. Then evaluate that series to express the number as a ratio of integers.
   (b) Use series to show that $0.\overline{9} = 1.\overline{0}$.

3. Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ converges. Find the sum of the series for those values of x. (Note: this is the geometric series with $r = x/3$ starting at $n = 1$.)

4. Find the value of r if $\sum_{n=2}^{\infty} r^n = 2$

HOMEWORK DAY 30 — Due Thursday, November 1st

Online: §11.4: 1,2,3,4,13.

§11.4: 10,16,18

HOMEWORK DAY 31 — Online only

Online: §11.5: 1,2,4,7,17,19,32.

HOMEWORK DAY 32 — Due Thursday, November 8th

Online: §11.6: 1,3,9,12,15.

1. Determine whether the following series converge or diverge, using any of the tests we have learnt about. If they converge, do they converge absolutely or conditionally? In each case, give a concise answer stating the test that you used and an explanation. *For example*:

   For problem 11.6:12 in the online hw you could write "Conv abs by direct comparison test, since $|a_n| = |\sin(4n)/4^n| \leq 1/4^n$, and $\sum 1/4^n$ is a converging p-series, so $\sum |a_n|$ converges")

   For problem 11.6:9 you could write "diverges by divergence test since the exponential 1.1^n grows faster than the algebraic n^4, so $|a_n|$ does not approach 0.")

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{\sqrt{k+4}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

   (c) $\displaystyle\sum_{n=3}^{\infty} \frac{4^n}{5^n-1}$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

(e) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^3+1}$

(f) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

(g) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$

2. Which grows faster as $n \to \infty$, the exponential function 100^n or the factorial function $n!$. Explain.

3. Explain why, if $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

4. §11.5, # 23 (approximate an alternating series to within a prescribed error)

HOMEWORK DAY 33 — Due Thursday, November 8th

Online: §11.8: 3,7,11,16,19.

1. Which of the following are power series?

(a) $\sum_{n=0}^{\infty}(3x)^n$ (b) $\sum_{n=0}^{\infty} \sqrt{x}^n$ (c) $\sum_{n=0}^{\infty}(x+2)^{2n}$

2. Determine the radius and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}}$$

HOMEWORK DAY 34 — Due Thursday, November 15th

Online: §11.9: 3,5,15.

1. Use the geometric series to find a power series representation centered at $x = 0$ and its radius of convergence, for

(a) $f(x) = \frac{1}{4+2x^2}$

(b) $f(x) = \tan^{-1}(2x)$

2. (a) Find a power series representation of $f(x) = \frac{1}{8+x}$. Find its radius of convergence.

(b) Use differentiation to find a power series representation of $f(x) = \frac{1}{(8+x)^2}$. State its radius of convergence.

(c) Find a power series representation of $f(x) = \frac{1}{(8+x)^3}$. State its radius of convergence.

(d) Find a power series representation of $f(x) = \frac{x^2}{(8+x)^3}$. State its radius of convergence.

HOMEWORK DAY 35 — Due Thursday, November 15th

Online: §11.10a: 3,5,6,13,16.

1. Find the Taylor series for f about $x = 4$ if

$$f^{(n)}(4) = \frac{(-1)^n n!}{3^n(n+1)}$$

   What is its radius of convergence?

2. Find the Maclaurin series for $f(x) = \cosh(x)$.

3. Find the Taylor series for $f(x) = \sqrt{x}$ centered at $x = 4$.

4. Find the Taylor series for $f(x) = x - x^3$ about $x = 2$.

HOMEWORK DAY 36 — Due Thursday, November 15th

§11.10b: 51.

Find the first 5 nonzero terms of the Maclaurin series in the following problems. For # 38,48, also find a formula for the nth term and write out the Maclaurin series using summation notation.

§11.10: 30,31,33,38,48.

HOMEWORK DAY 37 — Due Tuesday Nov 20th

Online: §11.11: 3,4,5,10.

1. Find the first 5 nonzero terms of the Taylor series of $f(x) = \sin x$ at $a = \pi/6$. State the linear approximation of f about $a = \pi/6$. State the Taylor polynomial of degree 2 p_2 for f about $a = \pi/6$. How large is the magnitude of the error in the approximation $f(x) \approx p_2(x)$, for $x \in [0, \pi/3]$, at most?

HOMEWORK DAY 38 — Due Tuesday Nov 20th

No Online.

1. *A simple pendulum.* An idealized simple pendulum is given by a mass m hanging from a massless string of length L. Its motion is described by the angle $\theta(t)$, where t is time. The distance travelled by the pendulum is the arclength $s(t) = L\theta(t)$. According to Newton's 2nd law, the pendulum mass×acceleration equals the restoring force F_{net} acting on it.

$$mL\theta''(t) = F_{net}$$

   where $F_{net} = mg \sin\theta$ (see picture). For small angles one often uses the approximation $\sin\theta \approx \theta$

to replace this differential equation by the simpler equation

$$mL\theta''(t) = mg\theta$$

(which is easy to solve, as you'll find in Math 316). Question: If the angle swings with $-\pi/10 \le \theta \le \pi/10$, what is an upper bound for the error made in the approximation

$$\sin\theta \approx \theta \ ?$$

2. (§12.11, # 33) An electric dipole consists of two electric charges of equal magnitude and opposite signs. If the charges are q and $-q$ and are located at a distance d from each other, then the electric field E at the point P in the figure is

$$E = \frac{q}{D^2} - \frac{q}{(D+d)^2}$$

By expanding this expression for E as a series in powers of d/D, show that E is approximately proportional to $1/D^3$ when P is far away from the dipole.

HOMEWORK DAY 41 — Online Only

Online: App H 1,2,3,5,6,7,8,10,11,13,15,17

HOMEWORK DAY 42 — Due Monday Dec 3rd

Online: App H 19,20,21,41,42,43,44,45,46

1. Find the real and imaginary parts of the following numbers and plot them in the complex plane.

   (a) $z = e^{2-\pi i/4}$ (b) $z = e^{-1-i}$

2. By writing the individual factors on the left in exponential form, performing the needed operations, and finally changing back to Cartesian coordinates, show that

   (a) $i(1-\sqrt{3}i)(\sqrt{3}+i) = 2(1+\sqrt{3}i)$ (b) $(-1+i)^7 = -8(1+i)$

3. Write the following numbers in the form $re^{i\theta}$ and in the form $a+ib$:

   (a) $(1+i)^{20}$ (b) $(1-\sqrt{3}i)^5$ (c) $(2\sqrt{3}+2i)^5$

   (d) $(1-i)^8$ (e) $\dfrac{-2}{1+3i}$

4. Show that (a) $|e^{i\theta}| = 1$, (b) $\overline{e^{i\theta}} = e^{-i\theta}$.