6.6:61: $\displaystyle\int_0^{1/2} \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/6} u\,du$

$u = \sin^{-1}(x)$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx$

$$= \frac{u^2}{2}\Big|_{u=0}^{u=\pi/6}$$

$$= \frac{1}{2}\left(\frac{\pi}{6}\right)^2 = \frac{\pi^2}{72}$$

6.6:2

$\int \tanh(x)$.

(a) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$f(-x) = \frac{e^{-x} - e^{-(x)}}{e^{-x} + e^{-(-x)}}$$

$$= \frac{e^{-x} - e^{x}}{e^{-x} + e^{x}}$$

$$= \frac{-(e^x - e^{-x})}{e^{x} + e^{-x}} = -f(x).$$

$\Rightarrow f(x)$ is odd.

6.6: 2 cont ...

(b)

$$\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}}$$

$$= \lim_{x\to\infty} \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= 1$$

$$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x\to-\infty} \tanh(x)$$

$$= \lim_{x\to\infty} \tanh(-x)$$

$$= \lim_{x\to\infty} -\tanh(x)$$

$$= -\lim_{x\to\infty} \tanh(x)$$

$$= -1.$$

from the first part.

(c) Intercepts: $\tanh(0) = \frac{e^0 - e^0}{e^0 + e^0} = \frac{0}{2} = 0$,

$\Rightarrow$ $(0,0)$ an intercept

Horizontal Asymptotes:

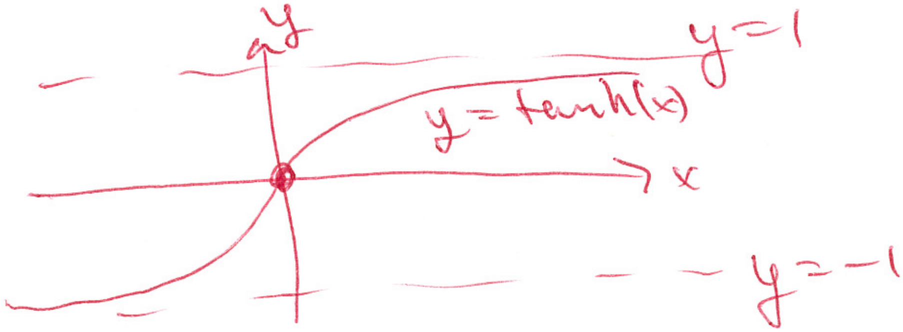
$$\lim_{x \to \pm\infty} \tanh(x) = \pm 1$$

$\Rightarrow$ $y = 1$ & $y = -1$ are HA

(d) $\frac{d}{dx}\left[\tanh(x)\right] = \frac{4e^{2x}}{(e^{2x}+1)^2} > 0$

$\Rightarrow$ always increasing

(e)



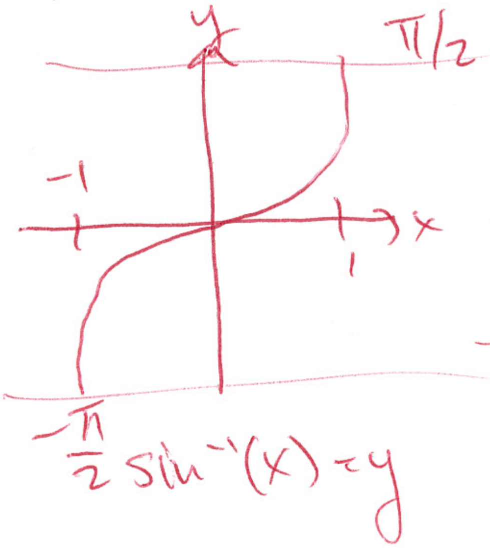
$$\operatorname{sech}(x)^2 = \left(\frac{2}{e^x + e^{-x}}\right)^2 = \frac{4}{((e^{2x}+1)e^{-x})^2}$$

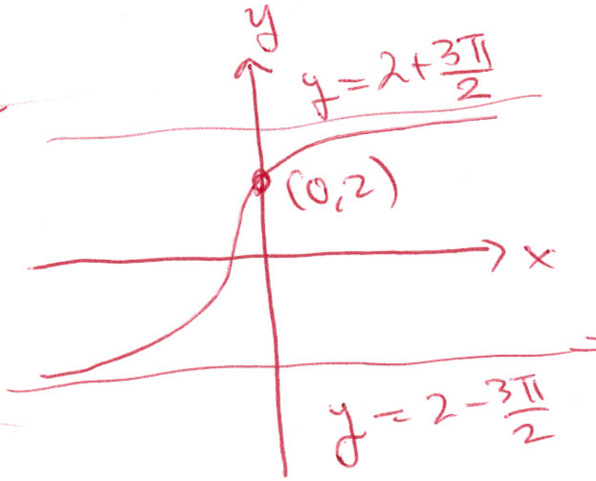
$$= \frac{4}{(1+e^{2x})^2(e^{-x})^2}$$

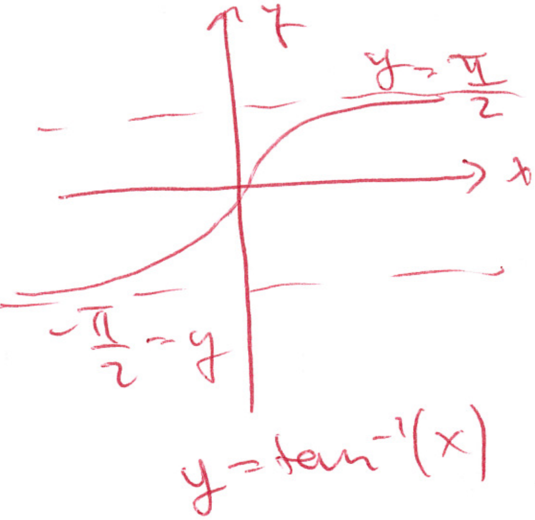
$$= \frac{4e^{2x}}{(1+e^{2x})^2}$$

$= \frac{d}{dx}\left[\tanh(x)\right]$ from part (d).

Day 6







§6.6:3: $y = x\sin^{-1}(x) + \sqrt{1-x^2}$

$$\frac{dy}{dx} = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$$

$$+ \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$= \sin^{-1}(x) + \frac{x}{\sqrt{1-x}} - \frac{x}{\sqrt{1-x^2}}$$

$$= \sin^{-1}(x)$$