Find the power series expansions of the indicated functions at the indicated centers. Please write out the full series (using summation notation) when appropriate. Many of these examples were done in class so your notes may be useful.

1 Basic

1. $f(x) = 1$ at $x_0 = 0$
2. $f(x) = 1$ at $x_0 = 100$
3. x at $x_0 = 0$
4. x at $x_0 = 2$
5. $1 + x^2 + x^3$ at $x_0 = 0$
6. $1 + x^2 + x^3$ at $x_0 = 1$
7. $f(x) = c + bx + ax^2$ at $x_0 = 0$ where a, b, c are constants.
8. $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ at $x_0 = 0$.

2 Geometric

1. (Geometric Series Formula) $\frac{1}{1-x}$ at $x_0 = 0$
2. The following can be derived from the Geometric Series Formula (1). (You don't need to apply the formula for Taylor Series in this case. They will always have a finite radius of convergence.)
   (a) $\frac{1}{1+x}$ at $x_0 = 0$
   (b) $\frac{1}{1-x^2} + 1 + x^2 + x^3$
   (c) $\frac{1}{1+x^2}$ at $x_0 = 0$
   (d) $\frac{1}{(1-x^2)}$ at $x_0 = 0$
   (e) $\frac{1}{1-x^2}$ at $x_0 = 0$
   (f) $\frac{x}{1-x^2}$ at $x_0 = 0$
   (g) $\frac{1}{1+x^l}$ at $x_0 = 0$, here l is a positive integer.
   (h) $\frac{1}{1+\frac{x-5}{4}}$ at $x_0 = 5$
   (i) $\frac{1}{1-x}$ at $x_0 = 5$ (Hint: use the following "factoring trick" $\frac{1}{1-x} = \frac{1}{-4-(x-5)} = \frac{-1}{4} \cdot \frac{1}{1+\frac{x-5}{4}}$)
   (j) $\frac{1}{1-x}$ at $x_0 = 6$
   (k) $\frac{1}{x}$ at $x = 10$. (Hint: use a "factoring trick" $\frac{1}{x} = \frac{1}{10+(x-10)}$ and finish like in problem (2i))
   (l) $\frac{1}{a+x^n}$ at $x_0 = 0$
   (m) $\frac{1}{1-x}$ at $x_0 = a$ (Hint: do the factoring trick again)
3. The following can be derived by taking derivatives and definite integrals of things you have already computed and maybe doing a little bit of extra manipulation.
   (a) $\frac{-1}{(1-x)^2}$ at $x_0 = 0$ (Hint: Take the derivative of $\frac{1}{1-x}$)
   (b) $\frac{-1}{(1-x)^2}$ at $x_0 = 0$ (Hint: Take the derivative of $\frac{1}{1-x}$ and multiply by (-1))
   (c) $\tan^{-1}(x)$ at $x_0 = 0$ (Hint: $\tan^{-1}(x) = \int_0^x \frac{dt}{1+t^2}$)
   (d) $\frac{1}{(1-x)^3}$ at $x_0 = 0$
   (e) $\frac{1}{(1-x)^n}$ at $x_0 = 0$
   (f) $-\ln(1-x)$ at $x_0 = 0$
   (g) $-\ln(1-x)$ at $x_0 = 0$ (use the formula for Taylor Series this time)

3 Exponential

1. e^x at $x_0 = 0$
2. The following can be derived using the known formula for the exponential function'
   (a) $e^x + 1 + x^2 + x^3$ at $x_0 = 0$
   (b) e^x at $x_0 = 2$
   (c) e^{2x} at $x_0 = 0$
   (d) e^{x^2} at $x_0 = 0$
   (e) $\text{erf}(x)$ at $x_0 = 0$ (The error function is defined by $\text{erf}(x) = \int_{-\infty}^{x} e^{-t^2} dt$.)
   (f) $\cosh(x)$ at $x_0 = 0$
   (g) $\sinh(x)$ at $x_0 = 0$
3. $\sin(x)$ at $x_0 = 0$
4. $\cos(x)$ at $x_0 = \pi/2$
5. $\cos(x) + \sin(x)$ (write out the first 6 terms) at $x_0 = 0$

4 Roots

1. $\sqrt{x}$ at $x_0 = 1$
2. $(1 + x)^{1/2}$ at $x_0 = 0$
3. $(1 + 2x)^{1/3}$ at $x_0 = 0$
4. $(1 + x)^a$ at $x_0 = 0$ where a is a postive number less than 1.
5. $\sqrt{x}$ at $x_0 = 1$ (using the formula of the previous problem) (Hint: $\sqrt{x} = \sqrt{1 + (x - 1)}$)

5 Manipulating Power Series

Pretty much just take any function "special function" off the street and it will be complicated...

1. You can multiply two power series together to get a new power series. Find the first three terms of
$$(\sum_{n=0}^{\infty} a_n x^n)(\sum_{n=0}^{\infty} b_n x^n)$$
.
2. Find the first three terms of the $e^x \cdot e^x$ expanded at $x_0 = 0$ by multiplying out the series
3. Find the first three terms of the series for $e^x\sqrt{1 + x}$ expanded at $x_0 = 0$.
4. If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ prove that $\frac{1}{1-x} \cdot f(x) = \sum_{n=0}^{\infty} S_n x^n$ where $S_n = \sum_{k=0}^{n} a_k$.