Math 31B — Homework 04 (Double Quiz)

Tuesday Quiz Date: October 29th
Thursday Quiz Date: October 31st

Part 1: Partial Fraction Expansion

1. 8.5: 3
2. 8.5: 5, 9 ,10

Part 2: Integrals with infinities (Improper integrals)

3. 8.6: 3,5,6
4. 8.6: 25, 26
5. 8.6: 37,39
6. (a) Compute $\int x^{-a} dx$ where $a \neq 1$.
   (b) For what values of $a > 0$ does the integral $\int_0^1 x^{-a} dx$ converge?
   (c) For what values of $a > 0$ does the integral $\int_1^\infty x^{-a}$ converge?
7. The Gamma function is defined by

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt, \qquad s > 0$$

   (a) Show that $\Gamma(1) = 1$
   (b) Using integration by parts with $u = t^{s-1}$ and $dv = e^{-t}dt$ show the reduction formula

$$\Gamma(s) = (s-1)\Gamma(s-1), \quad s > 1$$

   (c) Conclude that $\Gamma(n+1) = n!$ when n is a positive whole number. (Recall that $n! = n \cdot (n - 1) \cdot (n-2) \cdots 1$. For example $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

Part 3: Numerical Integration

8. 8.8: 1
9. 8.8: 20
10. 8.8: 45
11. (optional, but maybe needed for the next problem) In sage:
    (a) Write some code to do the left endpoint approximation of a function $f(x)$ with N intervals
    (b) Write some code to do the midpoint approximation of a function $f(x)$ with N intervals
    (c) Write some code to do Simpson's approximation of a function $f(x)$ with N intervals

    Try what your wrote out of some of your favorite functions to make sure it makes sense.
12. Define the function $f(x) = \int_0^x e^{-t^2} dt$.

(a) Find the values of $f(1), f(2), f(3), f(4), f(5), f(6), f(7), f(8), f(9)$ and $f(10)$ accurate to 4 decimal places by applying the left endpoint approximation with N intervals for various N.

(b) Do the same thing using the midpoint rule.

(c) Do the same thing using Simpson's approximation.

(A take home quiz problem may contain things similar to this exercise)

13. (optional) This problem is pretty tedious but those who care to know where Simpson's rule comes from should do it. I think it is good to do at least once in your life though as it explains where Simpson's rule comes from.

(a) Find the expression for the polynomial $p(x)$ of degree two passing through the points $(a, A), (b, B), (c, C)$.

(b) Let $p(x)$ be the polynomial passing through $(a, f(a)), (b, f(b)), (\frac{a+b}{2}, f(\frac{a+b}{2}))$ show that

$$\int_a^b p(x)dx = \frac{(b-a)}{6}\left[f(a) + 4f(\frac{a+b}{2}) + f(b)\right].$$