

Power Series Computations

1. Give a formula for the power series of $\sin(x)$ at $x_0 = 0$.
2. Give a formula for the power series of $\cos(x)$ at $a = 0$
3. Give a formula for the the power series of $\ln(1 - x)$ at $x_0 = 0$.
4. Give a formula for the power series of $\int e^{x^2} dx$ at $x = 0$ assuming the constant of integration is zero.
5. Give a formula for the power series of $\frac{1}{1-x}$ at $x_0 = 0$.
6. Give a formula for the power series of $\frac{1}{1-x}$ at $x_0 = 2$.
7. Give a formula for the power series of $\frac{1}{1+x^2}$ at $a = 0$.
8. 9.4: 4
9. 9.4: 9, 10
10. 9.4: 52

Error Estimates

11. How small must θ remain in order for the approximation

$$\sin(\theta) \approx \theta$$

to be accurate to 4 decimal places. (You don't need to be exact here, just do the best you can with the error estimates of the Taylor Series).

12. In 1748 Euler gave an approximation of e to 18 decimal places. Assuming that he used the formula

$$e = \sum_{n=0}^{\infty} \frac{1}{n!},$$

how many terms would he have needed to take in order to obtain this accuracy? (Do the best you can using the error estimates given in class.)