Math 31B — Homework 07

**Instructions** Compute the power series at the indicated points.

# 1 Polynomials

1. $f(x) = 1$ at $x_0 = 0$
2. $f(x) = 1$ at $x_0 = 100$
3. $x$ at $x_0 = 0$
4. $x$ at $x_0 = 2$
5. $1 + x^2 + x^3$ at $x_0 = 0$
6. $1 + x^2 + x^3$ at $x_0 = 1$
7. $f(x) = c + bx + ax^2$ at $x_0 = 0$ where $a, b, c$ are constants.
8. $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ at $x_0 = 0$.

# 2 Geometric

1. (Geometric Series Formula) $\frac{1}{1-x}$ at $x_0 = 0$
2. The following can be derived from the Geometric Series Formula (1). (You don't need to apply the formula for Taylor Series in this case. They will always have a finite radius of convergence.)
   (a) $\frac{1}{1+x}$ at $x_0 = 0$
   (b) $\frac{1}{1-x^2} + 1 + x^2 + x^3$
   (c) $\frac{1}{1+x^2}$ at $x_0 = 0$
   (d) $\frac{1}{(1-x^2)}$ at $x_0 = 0$
   (e) $\frac{1}{1-x^2}$ at $x_0 = 0$
   (f) $\frac{x}{1-x^2}$ at $x_0 = 0$
   (g) $\frac{1}{1+x^l}$ at $x_0 = 0$, here $l$ is a positive integer.
   (h) $\frac{1}{1+\frac{x-5}{4}}$ at $x_0 = 5$
   (i) $\frac{1}{1-x}$ at $x_0 = 5$ (Hint: use the following "factoring trick" $\frac{1}{1-x} = \frac{1}{-4-(x-5)} = \frac{-1}{4} \cdot \frac{1}{1+\frac{x-5}{4}}$)
   (j) $\frac{1}{1-x}$ at $x_0 = 6$
   (k) $\frac{1}{x}$ at $x = 10$. (Hint: use a "factoring trick" $\frac{1}{x} = \frac{1}{10+(x-10)}$ and finish like in problem (2i))
   (l) $\frac{1}{a+x^n}$ at $x_0 = 0$
   (m) $\frac{1}{1-x}$ at $x_0 = a$ (Hint: do the factoring trick again)
3. The following can be derived by taking derivatives and definite integrals of things you have already computed and maybe doing a little bit of extra manipulation.
   (a) $\frac{-1}{(1-x)^2}$ at $x_0 = 0$ (Hint: Take the derivative of $\frac{1}{1-x}$)

(b) $\frac{-1}{(1-x)^2}$ at $x_0 = 0$ (Hint: Take the derivative of $\frac{1}{1-x}$ and multiply by $(-1)$)

(c) $\tan^{-1}(x)$ at $x_0 = 0$ (Hint: $\tan^{-1}(x) = \int_0^x \frac{dt}{1+t^2}$)

(d) $\frac{1}{(1-x)^3}$ at $x_0 = 0$

(e) $\frac{1}{(1-x)^n}$ at $x_0 = 0$

(f) $-\ln(1-x)$ at $x_0 = 0$

(g) $-\ln(1-x)$ at $x_0 = 0$ (use the formula for Taylor Series this time)

# 3 Exponential

1. $e^x$ at $x_0 = 0$

2. The following can be derived using the known formula for the exponential function

   (a) $e^x - 1 - x^2 - x^3$ at $x_0 = 0$

   (b) $e^x$ at $x_0 = 2$

   (c) $e^{2x}$ at $x_0 = 0$

   (d) $e^{x^2}$ at $x_0 = 0$

   (e) $\text{erf}(x)$ at $x_0 = 0$ (The error function is defined by $\text{erf}(x) = \int_{-\infty}^x e^{-t^2} dt$.)

   (f) $\cosh(x)$ at $x_0 = 0$

   (g) $\sinh(x)$ at $x_0 = 0$

3. $\sin(x)$ at $x_0 = 0$

4. $\cos(x)$ at $x_0 = \pi/2$

5. $\cos(x) + \sin(x)$ (write out the first 6 terms) at $x_0 = 0$

# 4 Roots

1. $(1 + x)^{1/2}$ at $x_0 = 0$

2. $(1 + 2x)^{1/3}$ at $x_0 = 0$

3. $(1 + x)^a$ at $x_0 = 0$ where $a$ is a postive number less than 1.

4. $\sqrt{x}$ at $x_0 = 1$ (using the formula of the previous problem) (Hint: $\sqrt{x} = \sqrt{1 + (x - 1)}$)