Final

Math 264 — Fall 2010 — Dupuy

December 16, 2010

Do exactly six problems. Please clearly indicate which six of the problems you wrote down you wish to be graded on. The extra credit does not count as one of your six problems.

1. *Dot Products and Cross Products.* Let $\mathbf{v} = (1, -1, 1)$ and $\mathbf{u} = (0, 1, 0)$
   (a) Compute $\mathbf{v} \cdot \mathbf{u}$?
   (b) Compute $\mathbf{v} \times \mathbf{u}$?
   (c) What is the angle between $\mathbf{u}$ and $\mathbf{v}$? (You can leave your answer in terms of inverse trig functions)

2. *Planes*

   (a) What is a normal vector of the plane $x + y + z - 1 = 0$.

   (b) Find the plane containing the three points $(2, 0, 0), (0, 2, 0), (0, 0, 2)$.

3. *Lines*

(a) Find a parametrization of the line passing through the points $(-1, 0, 0)$ and $(0, 1, 1)$.

(b) Find a line parallel to the line $\mathbf{l}(t) = (1, 2, 3)t + (0, 2, 0)$ that passes through the point $(1, 1, 1)$.

4. *Parametric Curves* Let $\mathbf{a}(t) = (t+1, 1, (t+1)^2)$ and let $\mathbf{b}(s) = (s, s^2, 1)$

(a) Find where the curves parametrized by $\mathbf{a}(t)$ and $\mathbf{b}(t)$ intersect.

(b) Find the line tangent to $\mathbf{a}(t)$ at the point $(0, 1, 0)$.

5. *Tangent Planes*

(a) Find the plane tangent to the graph of $f(x, y) = x^2 + e^y$ at the point $(1, 0, 2)$.

(b) Find the plane tangent to the surface $x^2 + y^3 + z^4 = 2$ are the point $(1, 1, 0)$.

6. *Optimization*

(a) Find the extrema of the function $f(x, y) = x^2 - 2x + 1 - y^2 + 3$ and classify them using the second derivative test.

7. *Line Integrals.* Let L be the line segment parametrized by $\mathbf{r}(t) = (t, t, t)$ for $t \in [0, 1]$ and let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

(a) Compute the arclength of L,

$$\int_L ds$$

(b) Compute the line integral

$$\int_L \mathbf{F} \cdot \mathbf{T} ds$$

8. *Multiple Integrals*

(a) Compute

$$\int_0^1 \int_0^2 \int_0^1 e^x y z^2 dx dy dz.$$

(b) Let T be the tetrahedron bounded by the plane $x + y + z = 1$ in the first octant. Compute $\iint_T x dV$.

9. *Fundamental Theorem of Line Integrals*

(a) Find a potential for the vector field $\mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k}$,

(b) Let C be any continuous oriented curve starting at $(0, 0, 0)$ and ending at $(1, 2, 1)$, compute

$$\int_C \mathbf{F} \cdot \mathbf{T} ds.$$

10. *Green's Theorem.* Let $\mathbf{F} = -y\mathbf{i} + x\mathbf{j}$, and let C be the boundary of the unit square with vertices $(0,0),(1,0),(1,1),(0,1)$ oriented in the counter-clockwise direction, compute

$$\iint_C \mathbf{F} \cdot \mathbf{dr}$$

11. Extra Credit Problems:

(a) Compute $\int_{-\infty}^{\infty} e^{-x^2} dx$.

(b) Compute $\int_0^{\infty} \frac{\sin(x)}{x} dx$.

(c) Prove the Fundamental Theorem of Line Integrals: Let $u(x, y, z)$ be a scalar valued function such that $\nabla u = \mathbf{F}$ and C be an oriented curve starting at P and ending at Q, then

$$\int_C \mathbf{F} \cdot \mathbf{T} ds = u(Q) - u(P).$$

(Hint: You will need to use the fundamental theorem of calculus, a version of the chain rule and)

(d) Using the Jacobian determinant that $dV = \rho^2 \sin(\phi) d\rho d\phi d\theta$.