Test 3 — Math 264 — Fall 2009

December 7, 2009

1. Let D be the rectangle bounded by the lines $x = 0, x = 1, y = 0$ and $y = 2$. Compute

$$\int_D x^2 y + e^z y dA.$$

2. Find the volume region of the under surface $z = xy$ lying above the region bounded by $y = -x^2 + 1$ and $y = 0$ in the xy-plane.

3. Compute $\int_C \vec{F} \cdot \vec{T} ds$ where $\vec{F}(x, y, z) = x\hat{i} + y\hat{j} + z^2\hat{k}$, and C is parametrized by $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t\hat{k}$ where $t \in [0, 1]$.

4. Let $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$. Compute its potential and use it to evaluate $\int_C \vec{F} \cdot \vec{T} ds$ where C is parametrized by $\vec{r}(t) = \sin(t)\hat{\imath} + t\hat{\jmath} + t\sin(t)\hat{k}$ where $t \in [0, \pi/2]$.

5. Find the surface area $z = 4 - x^2 - y^2$ above the region $z = 0$.

6. Let C be the square contour made from line segments going from $(0,0)$ to $(1,0)$ to $(1,1)$ to $(0,1)$ back to $(0,0)$. Use Green's Theorem to evaluate the line integral

$$\int_C y^2 e^x dx + xe^y dy.$$

7. Let $\vec{F}$ be the vector field $\vec{F}(x,y) = A(x,y)\vec{\imath} + B(x,y)\vec{\jmath}$ and $u = u(x,y)$ be a function of two variable. Prove that

$$\nabla \cdot (u(x,y)\vec{F}(x,y)) = \nabla u(x,y) \cdot \vec{F}(x,y) + u(x,y)(\nabla \cdot \vec{F}(x,y)).$$

NOTE: $\nabla \cdot \vec{F} = \text{div}(\vec{F})$ so in other notation the problem says $\text{div}(u\vec{F}) = \nabla u \cdot \vec{F} + u\text{div}(\vec{F})$.

8. For $x = r\cos(\theta)$ and $y = r\sin(\theta)$ show that $\dfrac{\partial(x,y)}{\partial(r,\theta)} = r$ and conclude that $dA = r dr d\theta$.

9. Compute

$$\iiint_\Omega z dV$$

where Ω is the upper half of the unit ball, bounded above by $x^2 + y^2 + z^2 = 1$ and bounded below by $z = 0$. (You may want to convert to Spherical)

10. Determine if the vector field $\vec{F} = x\hat{i} + y^2\hat{j} + z\hat{k}$ is conservative.