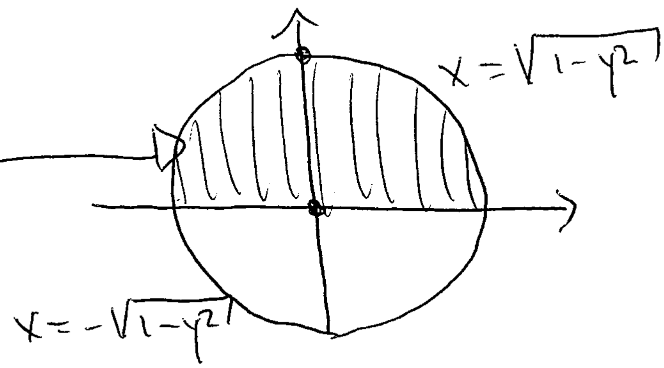
1. $\iint_D dA = \pi R^2$ (it's just the area!)

---

2. $$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

(a) $x \in [-\sqrt{1-y^2}, \sqrt{1-y^2}]$
$y \in [0,1]$



D domain of integration.

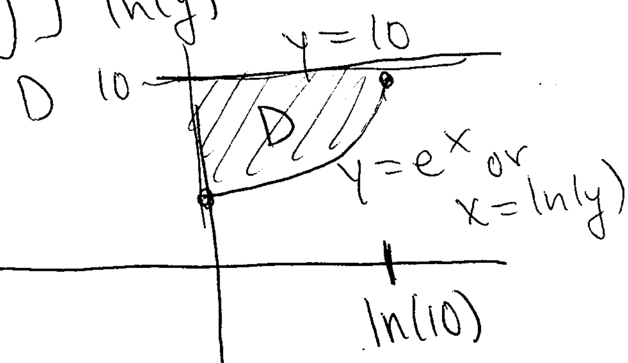
(b) $$\cancel{\iint_D dA} = \iint_D f(x,y)\,dA = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} f(x,y)\,dA$$

$y \in [0, \sqrt{1-x^2}]$
$x \in [-1,1]$

---

3. $$\int_0^{\ln(10)} \int_{e^x}^{10} \frac{1}{\ln(y)}\,dy\,dx = \iint_D \frac{1}{\ln(y)}\,dA$$

$y \in [e^x, 10]$
$x \in [0, \ln(10)]$



3 cont...

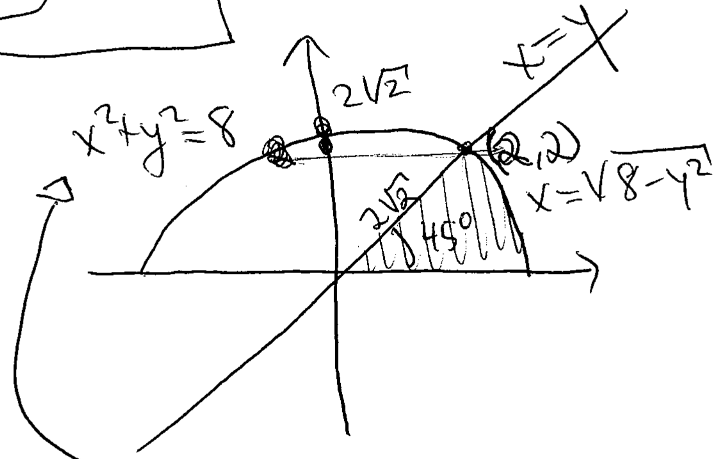
$x \in [0, \ln(y)]$

$y \in [1, 10]$

$$\iint_D \frac{1}{\ln(y)}\, dA = \int_1^{10} \int_0^{\ln(y)} \frac{1}{\ln(y)}\, dx\, dy$$

$$= \int_1^{10} dy = 9.$$

4. $\begin{cases} x \in [y, \sqrt{8-y^2}], \\ y \in [0, 2], \end{cases}$



when $x = y$ ~~whe~~ we get

$$2x^2 = 8$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = \pm 2$$

since $x > 0 \Rightarrow x = 2$.

In polar coordinates,

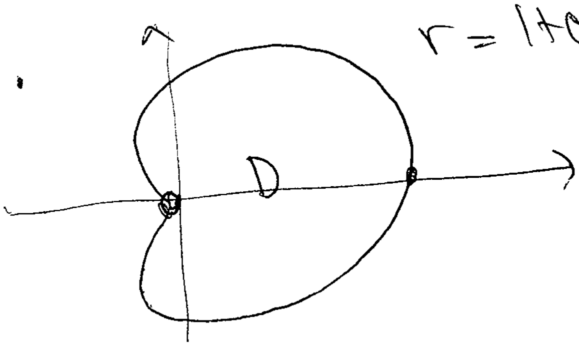
$\begin{cases} \theta \in [0, \pi/4] \\ r \in [0, 2\sqrt{2}] \end{cases}$

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\, dx\, dy = \int_0^{2\sqrt{2}} \int_0^{\pi/4} r^2\, d\theta\, dr$$

$$= \frac{\pi}{4} \cdot \frac{(2\sqrt{2})^3}{3}$$

$$= \frac{\pi}{\cancel{4}} \cdot \frac{\overset{2}{\cancel{8}} \cdot 2\sqrt{2}}{3} = \frac{4\pi\sqrt{2}}{3}.$$

5. $r = 1+\cos(\theta)$



$$\iint_D dA = \int_0^{2\pi}\int_0^{1+\cos(\theta)} r\,dr\,d\theta$$

$$= \int_0^{2\pi}\left(\frac{r^2}{2}\Bigg|_{r=0}^{r=1+\cos\theta}\right)d\theta$$

$$= \int_0^{2\pi}\left(1+\cos(\theta)\right)^2 d\theta$$

$$= \int_0^{2\pi}\left(1+2\cos(\theta)+\cos(\theta)^2\right)d\theta$$

$$= 2\pi + 0 + 4\pi$$

$$= 6\pi$$

FACT: $\int_0^{\pi/2}\cos(\theta)^2\,d\theta = \frac{\pi}{2}.$

---

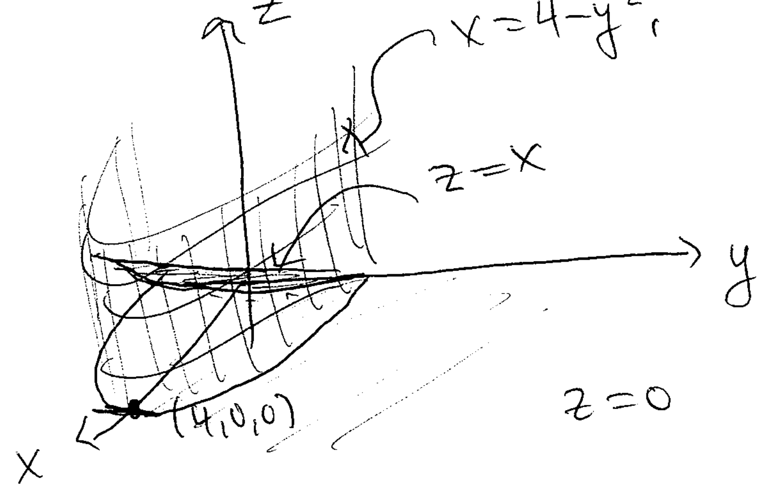
6. $$\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} x e^{yz} z^4\,dy\,dz\,dx$$

$$\overset{\text{Fub}}{=} \int_{-2}^{2}\int_{-3}^{3} e^{yz} z^4 \left\{\cancelto{0}{\int_{-1}^{1} x\,dx}\right\} dy\,dz$$

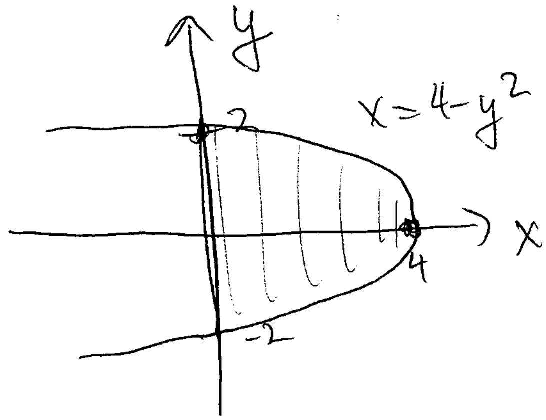
$$= 0$$

4

7. $\begin{cases} x = 4-y^2 \\ z = 0 \\ z = x \end{cases}$



$z \in [0, x]$



$x \in [0, 4-y^2]$
$y \in [-2, 2]$

$$\Rightarrow \text{vol} = \iiint_E dV$$

$$= \int_{-2}^{2} \int_{0}^{4-y^2} \int_{0}^{x} dz\,dx\,dy$$

$$= \int_{-2}^{2} \int_{0}^{4-y^2} x\,dx\,dy$$

$$= \int_{-2}^{2} \frac{x^2}{2} \Big|_{x=0}^{x=4-y^2} dy$$

$$= \frac{1}{2}\int_{-2}^{2} (4-y^2)^2\, dy \quad \cancel{\frac{1}{2}\int_{-2}^{2} \frac{d}{dy}(4-y^2)}$$

$$= \frac{1}{2}\int_{-2}^{2} (16 - 8y^2 + y^4)\, dy$$

$$= \int_{0}^{2} (16 - 8y^2 + y^4)\, dy$$

$$= 32 - 8\,\frac{2^3}{3} + \frac{2^5}{5}$$

$$= 32 - \frac{64}{3} + \frac{32}{5} = 32\left(\frac{1}{3} + \frac{1}{5}\right)$$

$$= 32\left(\frac{8}{15}\right)$$

$$= \frac{240+16}{15}$$

$$= \frac{256}{15}.$$

---

6. $D = \{(x,y) \mid 0 \le x \le 1 \;\&\; 0 \le y \le 1\}$

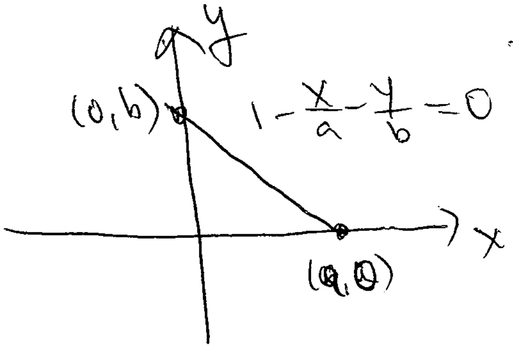
$f(x,y) = 100 - x - 2y, \quad f_x = -1, \; f_y = -2$

$$\iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA = \iint_D \sqrt{1+1+4}\, dA$$

$$= \sqrt{6} \iint_D dA$$

$$= \sqrt{6}.$$

10 cont ... $z \in [0, c(1 - \frac{x}{a} - \frac{y}{b})]$



$\Rightarrow y \in [0, b(1 - \frac{x}{a})]$
$x \in [0, a]$

$z = 0$

$$\text{vol}(T) = \iiint_T dV$$

$$= \int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} dz\, dy\, dx$$

$$= \int_0^a \int_0^{b(1-\frac{x}{a})} c\left(1 - \frac{x}{a} - \frac{y}{b}\right) dy\, dx$$

$$= c \int_0^a \int_0^{b(1-\frac{x}{a})} \left(1 - \frac{x}{a}\right) - \frac{y}{b}\, dy\, dx$$

$$= c \int_0^a \left(1 - \frac{x}{a}\right)^2 b \cancel{-\frac{1}{b}\left(\frac{b^2}{2}\right) dx} - \frac{1}{2b}\left(\left(1 - \frac{x}{a}\right)b\right)^2 dx$$

$$= \frac{cb}{2} \int_0^a \left(1 - \frac{x}{a}\right)^2 dx$$

$$= \frac{cb}{2} \int_0^a \frac{d}{dx}\left[-\frac{\left(1 - \frac{x}{a}\right)^3}{3} a\right] dx = -\frac{abc}{6} \int_0^a \frac{d}{dx}\left[\left(1 - \frac{x}{a}\right)^3\right] dx$$

$$= \frac{abc}{6}.$$

9. $$\mathrm{vol}(\text{sphere}) = \iiint_{B_R} dV$$

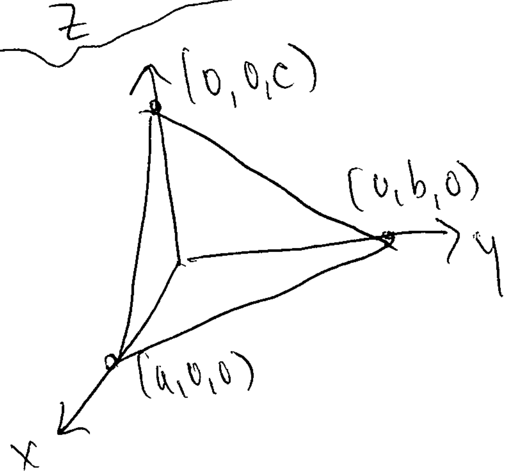
$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 \sin(\phi)\, dr\, d\phi\, d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin(\phi)\right)\left(\int_0^{R} r^2\, dr\right)$$

$$= (2\pi)\,(\;2\;)\left(\frac{R^3}{3}\right)$$

$$= \frac{4\pi R^3}{3}.$$

10.



PLANE:

$$Ax + By + Cz + D = 0$$

$$\Rightarrow \quad \begin{aligned} Cc + D &= 0 \\ Bb + D &= 0 \\ Aa + D &= 0 \end{aligned}$$

fix say $D = 1$ so we get $C = \frac{-1}{c}$, $B = -1/b$, $A = -1/a$,

$$\underbrace{1 - \frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0}_{\text{equation for plane passing through those points.}}$$