Test 3 — Math 264 — Fall 2010

December 9, 2010

Please show all of your work. You can finish the test a lot quicker if you think about the integrals before you dive in and start computing.

1. Let $D \subset \mathbb{R}^2$ be the disk centered at the origin of radius R. Compute $\iint_D dA$.

2. Evaluate the triple integral
$$\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} xe^{yz}z^4 dydzdx$$

3. The surface area of the graph of a function $f(x, y)$ above a domain D in the xy-plane is given by
$$S = \iint_D \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2}\ dA.$$
Find the surface area of the portion of the plane $x + 2y + z = 100$ that lies above the unit square $\{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$ in the first octant.

4. Consider the integral

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dxdy$$

(a) Sketch the domain of integration in $\mathbb{R}^2$.

(b) Change the order of integration.

5. Compute the integral

$$\int_0^{\ln(10)} \int_{e^x}^{10} \frac{1}{\ln(y)} dydx.$$

6. Evaluate the double integral:

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\,dx\,dy$$

7. Prove that the volume of a ball of radius R is equal to $\frac{4\pi R^3}{3}$.

Extra Credit (a) Let a, b and c be strictly positive real numbers numbers. Let H be the plane containing the points $(a,0,0)$,$(0,b,0)$ and $(0,0,c)$. Show that the tetrahedron bounded by H in the first octant has volume $\frac{abc}{6}$.

(b) Find the area of the region in the xy-plane enclosed by the curve

$$((x^2+y^2)-x)^2 = x^2+y^2$$