Exam 02 — Dupuy — Math 264 — Fall 2010

Instructions Remember to show all your work to get full credit. Please leave answers in their exact form. This is a closed book test. You may not use a calculator. If you need extra paper let me know.

Name:

Section:

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
EC	10	
Total	100	

1. (10 points) Compute $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ of the following functions

   (a) $u(x, y) = e^{xy}$.

   (b) $u(x, y) = \sin(xy)y$.

2. (10 points) Show that the function $u(x,t) = f(x - ct) + g(x + ct)$ satisfies the wave equation.

3. (10 points)

(a) How can you show that a limit of a function of two variable doesn't exist?

(b) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$ doesn't exist.

4. (10 points) Let $f(x, y)$ be a function of two variables. Using the chain rule show that $f_r = \frac{f_x x + f_y y}{\sqrt{x^2+y^2}}$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

5. (10 points) Find the plane tangent to the graph of the following functions at the specified point.

   (a) $e^{xy} + x^2$, $P = (0, 1, 1)$

   (b) $g(x, y) = x^3 + y^3$, $Q = (-1, 1, 0)$.

6. (10 points) Find the plane tangent to the surface at the specified point:

(a) $x^4 + y^4 + z^4 = 1$ $P = (1, 0, 0)$.

(b) $z - x^3 - y^3 = 0$, $Q = (-1, 1, 0)$ (Hint: compare to part b of the previous problem).

7. (10 points) Find the critical points of $f(x, y) = -(x^2 - 1)^2 - y^2$ and classify them.

8. (10 points) Find the maximal volume for a rectangle inscribes in the elliptsoid $x^2 + 2y^2 + 3z^2 = 1$.

9. (10 points)

(a) Plot the level curves of the function $f(x, y) = x^2 y$.

(b) What do the level surfaces of the function $g(x, y, z) = z^2 + 4y^2 + z^2$ look like? (a one sentence answer is acceptable).

10. Suppose that $f(x, y)$ is a differentiable function and $c \in \mathbb{R}$. Suppose $\vec{r}(t)$ is differentiable and parametrized the level set $\{(x, y) : f(x, y) = c\}$ Show that for all t in the domain of $\vec{r}(t)$ that $\vec{r}'(t)$ and $\nabla f(\vec{r}(t))$ are perpendicular.

11. (10 points) (EXTRA CREDIT)

(a) For $\vec{r} : \mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{r}(s,t) = (x(s,t), y(s,t))$ what is the Jacobian matrix of $\vec{r}$?

(b) How does this provide a way to view the chain rule of several variables as a genuine "chain rule" (write the pair of equations that we call the chain rule as a single matrix equation involving the Jacobian matrix).

(c) Show tha the Hessian matrix is the Jacobian matrix of the gradient.