Exam 2 — Dupuy — Spring 2010

Instructions Remember to show all your work to get full credit. Please leave answers in their exact form. This is a closed book test. You may not use a calculator. If you need extra paper let me know.

This has been retyped from an old exam given in MATH264 at University of New Mexico where the students had an hour and a half but could stay after to finish if they needed. Please note that the second exam in MATH121 at University of Vermont will contain double integrals.

Name:

Section:

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. (10 points) Compute $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for

   (a) $f(x, y) = e^{xy}$

   (b) $f(x, y) = x^3 + 4xy + y^2$

2. (10 points) Find the plane tangent to the graph of

$$f(x, y) = x^2 + 2x - y^2 + 6xy$$

at the point $(1, 1)$.

3. (10 points) Compute $D_{\vec{v}} f(x_0, y_0)$ in the following cases. If $\vec{u}$ is not initially given as a unit vector, please normalize it.

   (a) $f(x, y) = e^{x+y}$, $(x_0, y_0) = (1, 2)$, $\vec{u} = (1, 0)$

   (b) $f(x, y) = e^{x^2 y}$, $(x_0, y_0) = (1, 3)$, $\vec{u} = (1, 3)$

4. (10 points)

(a) What unit vector $\vec{u}$ maximizes $D_{\vec{u}}f(x_0, y_0)$?

(b) Explain/Prove that the statement you gave above is true.

(c) Find the direction of maximum increase of the function $x^2 + 2xy - 1$ at the point $(-1, 1)$.

5. (10 points) If $z = y + f(x^2 - y^2)$ where f is differentiable, show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$$

6. (10 points) Determine if the limit exists or not. If so, state the value of the limit.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3 + 4y^3}$.

(b) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^3 - y^3}{x - y}$.

7. (10 points) Compute $\dfrac{\partial f}{\partial s}$ and $\dfrac{\partial f}{\partial t}$ using the chain rule.

(a) $f(x,y) = e^{xy}$, $x = s\cos t$, $y = s\sin t$

(b) $f(x,y) = x^2 - y^2$, $x = e^{st}$, $y = e^{-t^2}$

8. (10 points) Find the critical points and determine if they are maxima, minima, or saddle points. (None existence is ok)

   (a) $f(x, y) = x^2 - 2y + 3$

   (b) $g(x, y) = x^2 - y^2 - 2x - 2y$

9. (10 points) Find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.
Hint: That surface is a constraint and you want to optimize the distance to the origin.
Tip: Optimize the distance squared rather than the straight up distance.

10. (10 points) Suppose that f is differentiable. Prove that the vectors tangent to the level sets

$$L_C = \{(x, y) : f(x, y) = C\}$$

are perpendicular to ∇f.
Hint: Suppose that $\vec{r}(t) = (x(t), y(t))$ parametrizes L_C.