**HOMEWORK 1** 13.1: 11, 16,
13.2: 22,
13.3: 26, ~~34~~ 38

**13.1 : 11** Find the equation of a sphere with radius 5 & center $(1,-4,3)$.

$$(x-1)^2 + (y+4)^2 + (z-3)^2 = 25.$$

**13.1 : 16.** Show the equation

$$x^2+y^2+z^2+8x-6y+2z+17=0$$

represents a sphere. Find its radius & center.

In the xz-plane $y=0$ so we get $(x-1)^2+(z-3)^2 = 25-16 = 9$.

$$x^2+8x = (x+4)^2-16,$$
$$y^2-6y = (y-3)^2-9,$$
$$z^2+2z = (z+1)^2-1,$$

$$\Rightarrow 0 = x^2+y^2+z^2+8x-6y+2z+17$$
$$= [(x+4)^2-16] + [(y-3)^2-9] + [(z+1)^2-1] + 17$$
$$= (x+4)^2 + (y-3)^2 + (z+1)^2 - 9.$$

$$\Rightarrow (x+4)^2 + (y-3)^2 + (z+1)^2 = 9$$

CENTER: $(-4,3,-1)$
RADIUS: 3

**13.2 : 22** : Find the magnitude & direction of $\vec{v} = (-4,2,4)$.

$$|\vec{v}| = \sqrt{16+4+16} = \sqrt{36} = 6,$$

13.2:22 cont...

Direction is $\vec{v}/|\vec{v}| = \frac{1}{6}(-4,2,4)$

$= (\frac{-2}{3}, \frac{1}{3}, \frac{2}{3})$.

13.3:26 For what values of $b$ are the vectors $(-6,b,2)$ & $(b,b^2,b)$ orthog?

$(-6,b,2)$, $(b,b^2,b)$ orthogonal

$\iff$ ~~$-6b$~~ $(-6,b,2)\cdot(b,b^2,b) = 0$.

$0 = (-6,b,2)\cdot(b,b^2,b)$

$= -6b + b^3 + 2b$

$= b(-4+b^2)$

$= b(b-2)(b+2)$

So the vectors are orthogonal when

$b = 0$, $b = -2$ or $b = +2$.

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13.3:34 If a vector has direction angles $\alpha = \pi/4$ & $\beta = \pi/3$, find the third.

Soln: $(\cos\alpha)^2 + (\cos\beta)^2 + (\cos\gamma)^2 = 1$

$\Rightarrow (1/\sqrt{2})^2 + (1/2)^2 + (\cos\gamma)^2 = 1$

$\Rightarrow (\cos\gamma)^2 = 1 - 3/4 = 1/4$

$\Rightarrow \cos\gamma = 1/2 \Rightarrow \gamma = \cos^{-1}(1/2) = \pi/4$. //

13.3: 38

Find the vector projection of $\vec{b} = (5, -1, 4)$ onto $\vec{a} = (-2, 3, -6)$.

$$|a| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

$$\vec{a} \cdot \vec{b} = (-2)(5) + (3)(-1) + (-6)(4) = -10 - 3 - 24 = -37$$

$$\text{(SCALAR PROJ)} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-37}{7}.$$

$$\text{(VECTOR PROJ)} = \frac{-37}{7}\left(\frac{\vec{a}}{|\vec{a}|}\right)$$

$$= \frac{-37}{7}\left(\frac{1}{7}(-2, 3, -6)\right)$$

$$= \frac{-37}{49}(-2, 3, -6).$$