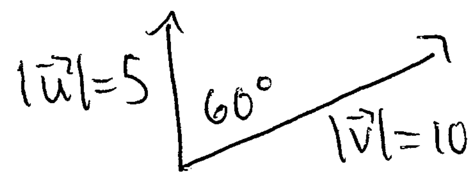
HOMEWORK 2

13.4: 14, 22
13.5: 4, 46
EXTRA PROBS #3

13.4 : 14

Find $|\vec{u} \times \vec{v}|$ and determine whether $\vec{u} \times \vec{v}$ is directed into or out of the page.



Soln.

- $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin(60°)$

$$= (5)(10)\left(\frac{\sqrt{3}}{2}\right) = 25\sqrt{3}.$$

- using the right hand rule we sweep $\vec{u}$ to $\vec{v}$ and see it is directed into the page.

13.4: 22

prove $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$ for all vectors $\vec{a}$ & $\vec{b}$ in $\mathbb{R}^3$.

pf. $(\vec{a} \times \vec{b}) \cdot \vec{b} = \vec{a} \cdot (\vec{b} \times \vec{b})$

$$= \vec{a} \cdot (\vec{0})$$

$$= 0. \blacksquare$$

13.5: 4

Find a line parallel to

$$\begin{cases} x = -1+2t \\ y = 6-3t \\ z = 3+9t \end{cases}$$

and passes through $(0, 14, -10)$.

Soln. Let's write our line in vector form,

$$\vec{r}(t) = (-1, 6, 3) + (2, -3, 9)t$$

our new line is then

$$\vec{p}(t) = \vec{r}_0 + (2, -3, 9)t$$

$$= (0, 14, -10) + (2, -3, 9)t. //$$

$\boxed{13.5:46}$ Where does the line through the points $(1, 0, 1)$ & $(4, -2, 2)$ intersect the plane

$$x + y + z = 6.$$

Soln.

First write down an expression for the line through the two prescribed points.

$$\vec{r}(t) = (1, 0, 1)t + (1-t)(4, -2, 2)$$

$$= (t, 0, t) + (4(1-t), -2(1-t), 2(1-t))$$

$$= (t + 4 - 4t, 0 + (-2) + 2t, t + 2 - 2t)$$

$$= (-3t + 4, 2t - 2, -t + 2).$$

Then intersect it with the plane,

$$x(t) + y(t) + z(t) = 6$$

$$\Rightarrow (-3t + 4) + (2t - 2) + (-t + 2) = 6$$

$\Rightarrow -2t+4=6 \Rightarrow t=-1.$

So when $t=-1$, the line intersects the plane.

$$\begin{aligned}\vec{r}(-1) &= (1,0,1)(-1) + (1-(-1))(4,-2,2)\\ &= (-1,0,-1) + (2)(+4,-2,2)\\ &= (-1+8,\ 0-2,\ -1+4)\\ &= (7,-2,3)\end{aligned}$$

point where they intersect.

QUATERNION & COMPLEX PROBLEM 3

$\vec{v} = i+j+K$

$\vec{w} = -i+2j$

(a) $$\vec{v} \times \vec{w} = \begin{vmatrix} i & j & K \\ 1 & 1 & 1 \\ -1 & 2 & 0 \end{vmatrix}$$

$$= i(-2) - j(1) + K(2+1)$$

$$= -2i - j + 3K. \,//$$

(b) $$\vec{v} \times \vec{w} = \frac{1}{2}(\vec{v}\vec{w} - \vec{w}\vec{v})$$

$$= \frac{1}{2}\big((i+j+K)(-i+2j) - (-i+2j)(i+j+K)\big)$$

$$= \frac{1}{2}\Big(\cancel{-i^2} - ji - ki + 2ij \cancel{+2j^2} + 2kj$$
$$\cancel{+i^2} + ij + ik - 2ji \cancel{-2j^2} - 2jk\Big)$$

$$= \frac{1}{2}\left(2k - 2j + 4k - 4i\right)$$

$$= -2i - j + 3k. //$$