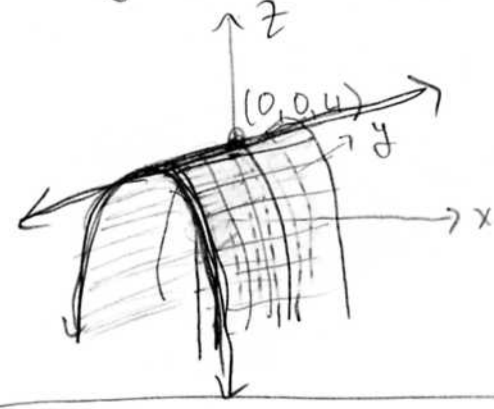
HOMEWORK 3

13.6: 4, 42, 44
14.1: 18, 42.

13.6

4. Describe & sketch $z = 4 - x^2$.

soln. The graph will be a parabolic cylinder.



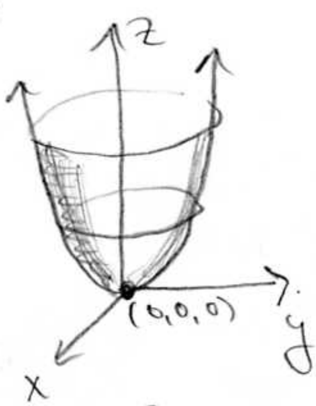
the traces along each $y = $ const slice will be parabolas of the form

$$z = 4 - x^2.$$

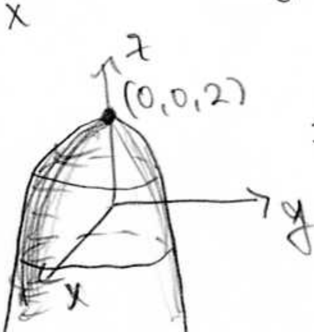
42. Sketch the region bounded by the parabolas

$$z = x^2 + y^2$$
$$\& \quad z = 2 - x^2 - y^2.$$

soln.



$z = x^2 + y^2$

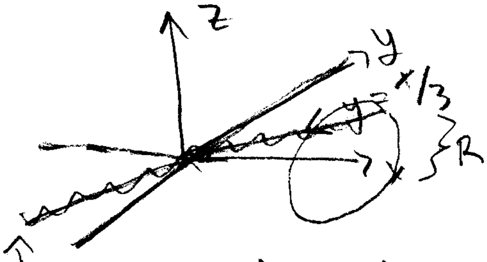


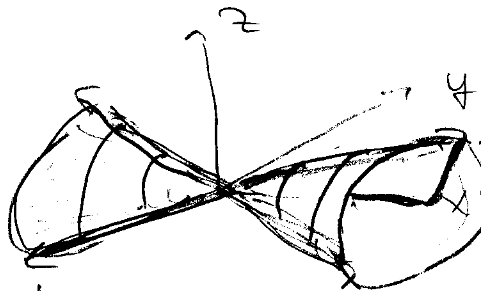
$z = 2 - x^2 - y^2$.

intersect in a circle

region bounded by both.

44. Find equation obtained by rotating line $x=3y$ about x-axis.





The distance from the x-axis is given by the y-coordinate (for a point in the xy-plane). Replacing this distance with the more general distance R,

$$x^2 - 9R^2 = 0. \qquad R^2 = y^2+z^2.$$

$$\Rightarrow \boxed{x^2 = 9(y^2+z^2)}$$

14.1

18. Find vector & parametric eqns for a line that passes through $P=(-2,4,0)$ & $Q=(6,-1,2)$

<u>soln.</u>

$$\vec{r}(t) = (1-t)P + tQ$$
$$= (1-t)(-2,4,0) + t(6,-1,2)$$
$$= (-2,4,0) + t(4,-5,2). \quad \underline{\text{vector}}$$

using $\vec{r}(t) = (x(t), y(t), z(t))$

$$\begin{cases} x(t) = -2+4t \\ y(t) = 4-5t \\ z(t) = 2t \end{cases} \quad \underline{\text{parametric}}$$

42. Do paths

$$\vec{r}_1(t) = (t, t^2, t^3)$$

&

$$\vec{r}_2(s) = (1+2s, 1+6s, 1+14s)$$

intersect ?

$$\vec{r}_1(t) = \vec{r}_2(s) \Rightarrow \begin{cases} t = 1+2s \\ t^2 = 1+6s \\ t^3 = 1+14s \end{cases}$$

$$\Rightarrow (1+2s)^2 = 1+6s \Rightarrow \cancel{1} + 4s + 4s^2 = \cancel{1} + 6s$$

$$\Rightarrow 4s^2 - 2s = 0$$

$$\Rightarrow 2s(2s-1) = 0$$

So $s=0$ or $s=1/2$ are the possible points where the curves intersect.

Correspondingly on the 1st curve we have

$$t = 1$$

or

$$t = 1+2(1/2) = 2.$$

Let's see if the third point is consistent:

- $\begin{cases} \text{If } t = 1 \text{ then } t^3 = 1. \\ \text{If } s = 0 \text{ then } 1+14s = 1. \end{cases}$

so, $\vec{r}_1(1) = \vec{r}_2(0)$

is a place where the paths intersect.

$\begin{cases} \text{If } t=2 \text{ then } t^3 = 8 \\ \text{If } s = 1/2 \text{ then } 1+14s = 8. \end{cases}$

So $\vec{r}_1(2) = \vec{r}_2(1/2)$

$\therefore$ The paths intersect at the points $(1,1,1)$ & $(2,4,8)$.

Also, since $s \neq t$ at any point of intersection, the particles never collide.