HOMEWORK 4

- 14.2: 10, 24
- 14.3: 16
- 14.4: 14, 34

14.2: 10. find the derivative of the vector function:

$$\vec{r}(t) = (\tan t, \sec t, 1/t^2).$$

$$\frac{d}{dt}[\vec{r}(t)] = (\sec(t)^2, \sec(t)\tan(t), -1/t^3).$$

14.2

24. Find the tangent line of curve at specified point

$$\begin{cases} x = e^t \\ y = te^t \\ z = te^{t^2} \end{cases} \qquad (1,0,0) = \vec{r}(0)$$

$\vec{r}(t) = (e^t, te^t, te^{t^2})$

$\vec{r}'(t) = (e^t, e^t + te^t, e^{t^2} + te^{t^2}(2t))$

$= (e^t, e^t(1+t), e^{t^2}(1+2t^2))$

$$\left.\begin{aligned} \vec{l}(s) &= \vec{r}(0) + s\vec{r}'(0) \\ &= (1,0,0) + s(1,1,1) \end{aligned}\right\} \text{tangent line.}$$

18.

14.3. Reparametrize the curve

$$\vec{r}(t) = \left(\frac{2}{t^2+1} - 1\right)\hat{i} + \left(\frac{2t}{t^2+1}\right)\hat{j}$$

in terms of arclength from the point (1,0).

Express reparametrization in simplest form. What can you conclude about the curve?

Soln: $(1,0) = \vec{r}(0)$

$\vec{r}' = \left(\frac{d}{dt}\left[\frac{2}{\ } \ \right], \frac{d}{dt}\left[\frac{2t}{\ } \right] \right)$

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since

$$\frac{d}{dt}\left[\frac{2}{t^2+1}-1\right] = 2\,\frac{-1}{(t^2+1)^2}\,\frac{d}{dt}\left[t^2+1\right]$$

$$= \frac{-4t}{(t^2+1)^2} = x'(t).$$

$$\frac{d}{dt}\left[\frac{2t}{t^2+1}\right] = \frac{2(t^2+1) - 2t(2t)}{(t^2+1)^2}$$

$$= \frac{2t^2+2-4t^2}{(t^2+1)^2} = y'(t).$$

$$\Rightarrow |r'(t)|^2 = x'(t)^2 + y'(t)^2$$

$$= \left[\frac{-4t}{(t^2+1)^2}\right]^2 + \left[\frac{-2t^2+2}{(t^2+1)^2}\right]^2$$

$$= \frac{1}{(t^2+1)^4}\left[16t^2+4t^4-8t^2+4\right]$$

$$= \frac{4}{(t^2+1)^4}\left[t^4+2t^2+1\right]$$

$$= \frac{4}{(t^2+1)^2}$$

$$\Rightarrow s = \int_0^t |r'(\tau)|\,d\tau = 2\int_0^t \frac{d\tau}{\tau^2+1} = 2\tan^{-1}(t).$$

Reparametrizing gives

$$\vec{p}(s) = \left(\frac{2}{\tan(s/2)^2+1} - 1,\ \frac{2\tan(s/2)}{\tan(s/2)^2+1} \right)$$

using the identities

- $(\tan\theta)^2 + 1 = (\sec\theta)^2$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$$\cos(2\theta) = (\cos\theta)^2 - (\sin\theta)^2$$
$$= 2(\cos\theta)^2 - 1$$

$\Rightarrow$ • $\dfrac{\cos(2\theta)+1}{2} = (\cos\theta)^2.$

with $\theta = s/2$ we get

$$\frac{2}{\tan(s/2)^2+1} - 1 = \frac{2}{\sec(\frac{s}{2})^2} - 1$$
$$= 2\cos(\tfrac{s}{2})^2 - 1$$
$$= 2\left(\frac{\cos(s)+1}{2}\right) - 1$$
$$= \cos(s)$$

$$\frac{2\tan(s/2)}{\tan(s/2)^2+1} = \frac{2\tan(s/2)}{\sec(s/2)^2}$$

$\therefore \; \vec{p}(s) = (\cos(s), \sin(s)),$

which means the origional parametrization was just a different parametrization of the circle.

14.4.

14. Find velocity, accel, & speed.

$$\vec{r}(t) = (t\sin t, t\cos t, t^2)$$

soln

VELOCITY:

$\vec{v}(t) = \vec{r}\,'(t) = (\sin(t) + t\cos(t), \cos(t) - t\sin(t), 2t)$

SPEED: $s(t) = |\vec{v}(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$

$= \left[\sin(t)^2 + \cancel{2t\sin(t)\cos(t)} + \underline{t^2\cos(t)^2} + \underline{\cos(t)^2} - \cancel{2t\cos(t)\sin(t)} + \underline{t^2\sin(t)^2} + 4t^2\right]^{1/2}$

$= \left[1 + t^2 + 4t^2\right]^{1/2} = \sqrt{1+5t^2}.$

ACCEL:

$\vec{a}(t) = \vec{v}\,'(t)$

$= (\cos(t) + \cos(t) - t\sin(t), -\sin(t) - \sin(t) - t\cos(t), 2)$

$= (\cos(t) + \cos(t) - t\sin(t), -2\sin(t) - t\cos(t), 2).$

$= (2\cos(t) - t\sin(t), -2\sin(t) - t\cos(t), 2).$

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34. Find tangential & normal components of acceleration vector.

$$\vec{r}(t) = (1+t)\,i + (t^2 - 2t)\,j.$$

$\vec{v}(t) = \vec{r}\,'(t) = i + (2t-2)j$

#34 cont...

$$T(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{i + (2t-2)j}{\sqrt{1+(2t-2)^2}}$$

$$= \frac{i + (2t-2)j}{\sqrt{1+4t^2-8t+4}}$$

$$= \frac{i + (2t-2)j}{\sqrt{4t^2-8t+5}}$$

$(x,y) \perp (-y,x)$

since the curve is 2D we can use this trick.

$$\left[N(t) = \frac{-(2t-2)i + j}{\sqrt{4t^2-8t+5}} \right]$$

TANGENTIAL COMPONENT:

$$\vec{a}(t) \cdot T(t) = 2j \cdot \left(\frac{i + (2t-2)j}{\sqrt{4t^2-8t+5}} \right)$$

$$a_T(t) = \frac{2(2t-2)}{\sqrt{4t^2-8t+5}}.$$

NORMAL COMPONENT:

$$a_N(t) = \vec{a}(t) \cdot N(t) = 2j \cdot \left(\frac{-(2t-2)i + j}{\sqrt{4t^2-8t+5}} \right)$$

$$= \frac{2}{\sqrt{4t^2-8t+5}}$$

$$\vec{a}(t) = a_T(t)\,T(t) + a_N(t)\,N(t)$$