HOMEWORK 5

15.3 : 16
15.4 : 4
15.5 : 10
15.6 : 6, 23

15.3 : 16 Find first partials,

$$f(x,y) = x^4 y^3 + 8x^2 y.$$

$$\frac{\partial f}{\partial x} = 4x^3y^3 + 16xy$$

$$\frac{\partial f}{\partial y} = 3x^4y^2 + 8x^2$$

15.4 : 4 Find equation for plane tangent to graph of $f(x,y) = y\ln(x)$ at the point $(1,4,0)$.

$$\nabla f(x,y) = \left(\frac{y}{x}, \ln(x)\right).$$

$$\Rightarrow \vec{n} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right) = \left(\frac{y}{x}, \ln x, -1\right)$$

at the point $(1,4,0)$ this becomes,

$$\vec{n} = (4, 0, -1).$$

$$\begin{aligned} 0 &= \vec{n}\cdot(\vec{r} - \vec{r}_0) \\ &= (4,0,-1)\cdot(x-1, y-4, z-0) \\ &= 4(x-1) + 0(y-4) - z \\ &= 4(x-1) - z. // \end{aligned}$$

15.5:10 Apply the chain rule,

$z = e^{x+2y}$, where $x(s,t) = s/t$
$y(s,t) = t/s$

- $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$
$$= \left[e^{x+2y}\right]\frac{\partial}{\partial s}\left[s/t\right] + \left[2e^{x+2y}\right]\frac{\partial}{\partial s}\left[t/s\right]$$
$$= e^{x+2y}\cdot(1/t) + 2e^{x+2y}(-t/s^2)$$
$$= e^{\frac{s}{t}+2\frac{t}{s}}\left(\frac{1}{t} + \frac{-2t}{s^2}\right).$$

- $$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$
$$= \left(e^{x+2y}\right)\frac{\partial}{\partial t}\left[s/t\right] + \left(2e^{x+2y}\right)\frac{\partial}{\partial t}\left[t/s\right]$$
$$= \left(e^{x+2y}\right)\left(-s/t^2\right) + \left(2e^{x+2y}\right)(1/s)$$
$$= e^{\frac{s}{t}+2\frac{t}{s}}\left(\frac{-s}{t^2} + \frac{2}{s}\right).$$

15.6:6 Find the directional derivative of $f(x,y) = x\sin(xy)$ at $P=(2,0)$ in direction of angle $\theta = \pi/3$.

$$\vec{u} = \left(\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$$
$$= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right).$$

$$\nabla f = \left(\sin(xy) + xy\cos(xy),\ x^2\cos(xy)\right).$$

$$(\nabla f)(2,0) = \left(\sin(0) + (2)(0)\cos(0),\ 2^2\cos(0)\right)$$
$$= (0,4).$$

$$\therefore\ (D_{\vec{u}}f)(2,0) = (\nabla f)(2,0)\cdot\vec{u}$$
$$= (0,4)\cdot\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$
$$= 2\sqrt{3}.//$$

<u>15.6: 23</u> Find maximum rate of change of f and the direction it occurs at the given point.

<u>NOTE</u>: The direction for which the rate of change is maximum is the unit vector $\vec{u}$ such that $(D_{\vec{u}}f)(x_0,y_0)$ is the greatest.

Since

$$(D_{\vec{u}}f)(x_0,y_0) = (\nabla f)(x_0,y_0)\cdot\vec{u}$$

$(D_{\vec{u}}f)(x_0,y_0)$ will be largest when $\vec{u} = \dfrac{\nabla f(x_0,y_0)}{\|\nabla f(x_0,y_0)\|}$.

It just remains to compute:

for $f(x,y) = \sin(xy)$,

$$\nabla f = (y\cos(xy), x\cos(xy)).$$

$$\Rightarrow (\nabla f)(1,0) = (0,1).$$

* This is already a unit vector so we don't need to normalize.

$$\therefore \quad \vec{u} = (0,1) \quad \Big\} \ \underline{\text{Direction of greatest change}}$$

$$\& \quad (D_{\vec{u}}f)(0,1) = (\nabla f)(0,1)\cdot\vec{u} = 1. \quad \Big\} \ \underline{\text{Max Rate of Change}}$$