HOMEWORK 6

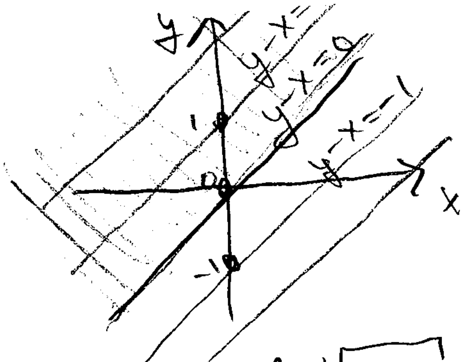
15.1:14, 15.2:14
15.7:4, 15.8:18, 15.8:33

15.1:14 Find & Sketch Domain

$$f(x,y) = \sqrt{y-x}\,\ln(y+x)$$

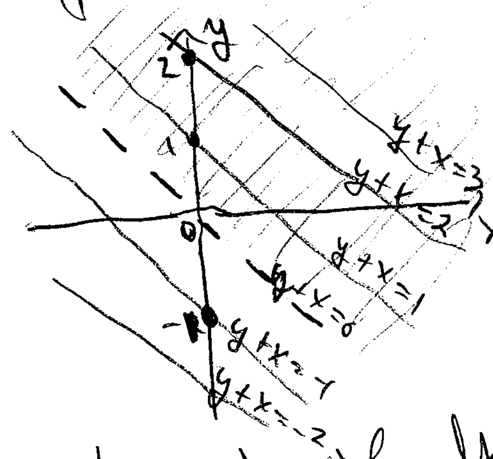
- $\sqrt{y-x}$ valid for $y-x \geq 0$
- $\ln(y+x)$ valid for $y+x > 0$

} break up into parts.



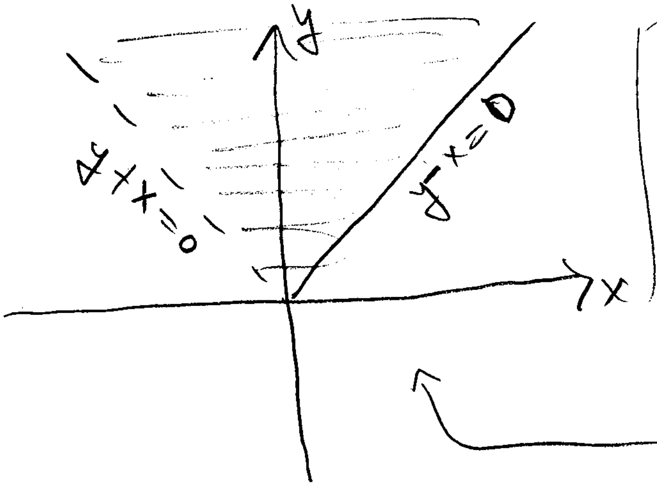
domain of $\sqrt{y-x}$

$\{(x,y)\in\mathbb{R}^2 : y-x \geq 0\} = D_1$



domain of $\ln(y+x)$

$\{(x,y)\in\mathbb{R}^2 : y+x > 0\} = D_2$



$$\boxed{\begin{aligned} \text{Domain} &= D_1 \cap D_2 \\ &= \{(x,y)\in\mathbb{R}^2 : y-x \geq 0 \ \& \ y+x > 0\}. \end{aligned}}$$

← Domain of $\sqrt{y-x}\,\ln(x+y)$.

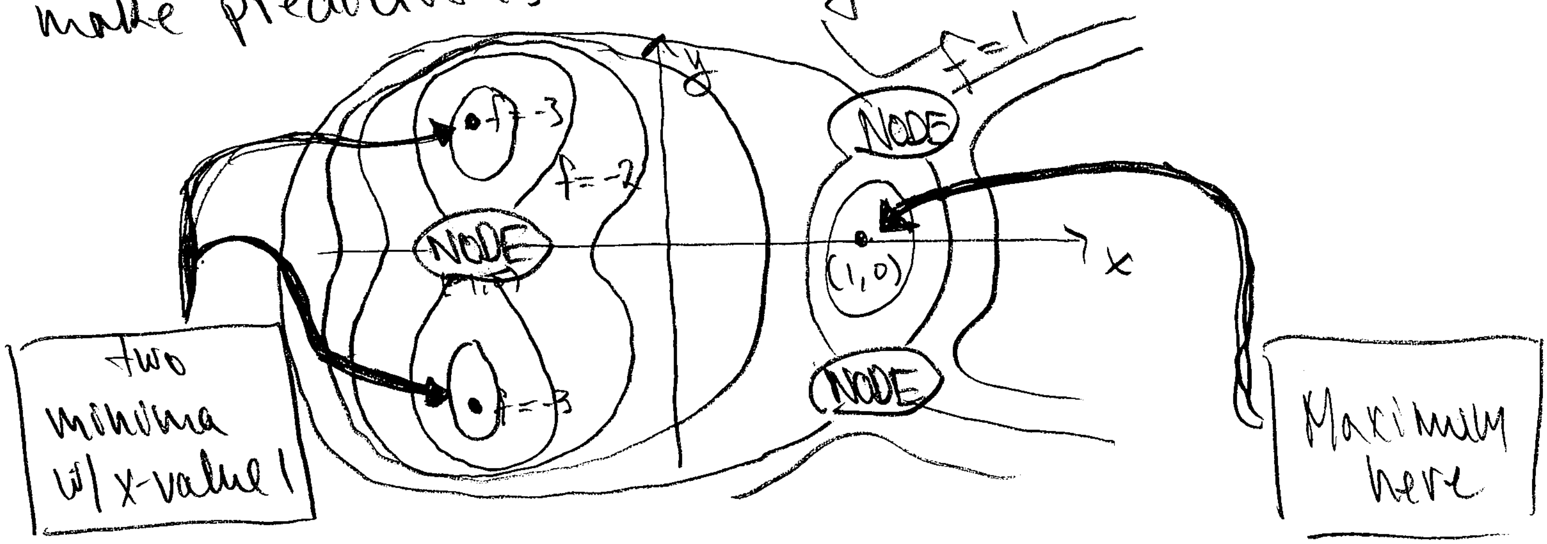
15.2:14

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}}$$

$$= \lim_{(x,y)\to(0,0)} x^2-y^2$$

$$=0.$$

15.7:4 Given graph of $f(x,y) = 3x - x^3 - 2y^2 + y^4$ make predictions & verify:



COMPUTATION:

$$\frac{\partial f}{\partial x} = 3 - 3x^2$$

$$\frac{\partial f}{\partial y} = -4y + 4y^3$$

$$0 = \frac{\partial f}{\partial x} = 3(1-x^2) = 3(1-x)(1+x)$$

$$0 = \frac{\partial f}{\partial y} = 4y(-1+y^2) = 4y(y-1)(y+1)$$

Critical Points:

(1,0)	(-1,0)
(1,1)	(-1,1)
(1,-1)	(-1,-1)

Now Apply 2nd Derivative test to see if they are max's or min's.

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= -6x \\ \frac{\partial^2 f}{\partial y^2} &= -4 + 12y^2 \\ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} &= 0 \end{aligned}\right\} \Rightarrow H = \begin{bmatrix} -6x & 0 \\ 0 & -4+12y^2 \end{bmatrix}$$

$$\det H = -6x(-4+12y^2)$$
$$= x(24-36y^2)$$

(x,y)	$\det H = x(24-36y^2)$	$f_{xx} = -6x$	RESULT
$(1,0)$	$24 > 0$	-6	max
$(1,1)$	$1(24-36) < 0$	not needed	NODE
$(1,-1)$	$1(24-36) < 0$	not needed	NODE
$(-1,0)$	$-24 < 0$	not needed	NODE
$(-1,1)$	$(-1)(24-36) > 0$	6	min
$(-1,-1)$	$(-1)(24-36(-1)^2) > 0$	6	min

Find extreme values of $f(x,y)$

15.8:18 ~~Optimize~~ $\begin{cases} f(x,y) = 2x^2+3y^2-4x-5 \\ x^2+y^2 \leq 16 \end{cases}$

$f_x = 4x-4$
$f_y = 6y$ $\Rightarrow$ $(x,y) = (1,0)$ a critical point inside the region.

$f(1,0) = 2-4 = -2$. Since $f_{xx} > 0$ $\Rightarrow$ concave up so $f(1,0) = -2$ is a minimum.

ON BOUNDARY: $x^2+y^2-16 = g(x,y)$.

$\frac{\partial g}{\partial x} = 2x$, $\frac{\partial g}{\partial y} = 2y$.

$$\nabla f = \lambda \nabla g \Rightarrow \begin{cases} \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \end{cases} \Rightarrow \begin{cases} 4x-4 = \lambda(2x) \\ 6y = \lambda(2y) \end{cases}$$

We need to solve:

$$\begin{cases} 4x-4 = 2\lambda x & (1) \\ 6y = 2\lambda y & (2) \\ x^2+y^2 = 16 & (3) \end{cases}$$

$(2) \Rightarrow (6-2\lambda)y = 0 \Rightarrow y=0$ or $\lambda = 3$.

if $\lambda = 3$: $(1) \Rightarrow 4x-4 = 12x \Rightarrow -8x-4=0$

$\Rightarrow -4(x+2) = 0 \Rightarrow \boxed{x=-2}$.

$(3) \Rightarrow y^2 = 16-4 \Rightarrow y^2 = 12 \Rightarrow \boxed{y = \pm 2\sqrt{3}}$ $\boxed{(-2, \pm 2\sqrt{3})}$

if $y=0$: $(3) \Rightarrow \boxed{x=4}$ $\boxed{(4,0)}$

$f(-2, 2\sqrt{3}) = 2(4) + 3(12) - 4(-2) - 5$

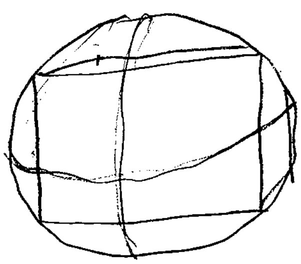
~~$f(-2, \pm 2\sqrt{3}) = 2(4) + 3(12) - 4(2) - 5$~~ $= 47$

~~$= 31.$~~ 47

$f(\pm 4, 0) = 2(16) - 4(4) - 5 = 11.$

$\therefore$ the **maximum** value of f on the region is at $(-2, 2\sqrt{3})$ where $f = 47.$ //

15.8:33 "Optimize volume of rectangle inscribed in sphere case $r=1$:



$$\begin{cases} x^2+y^2+z^2=1, & \begin{pmatrix}\text{Constraint} \\ \text{w/ } x,y,z \geq 0.\end{pmatrix} \\ V(x,y,z) = (2x)(2y)(2z) = 8xyz. \end{cases}$$

$$\nabla V = \lambda \nabla g \Rightarrow \begin{cases} 8yz = \lambda 2x & (1) \\ 8xz = \lambda 2y & (2) \\ 8xy = \lambda 2z & (3) \\ x^2+y^2+z^2=1 & (4) \end{cases}$$

$(1) \Rightarrow \lambda = \frac{4yz}{x}.$ $(2) \Rightarrow \cancel{8}x\cancel{z} = \left(\frac{\cancel{4}y\cancel{z}}{x}\right)\cancel{2}y$

$\Rightarrow x^2 = y^2.$ Since x & $y \geq 0 \Rightarrow x = y.$

$(3) \Rightarrow \cancel{8}x^2 = \left(\frac{\cancel{4}\cancel{y}z}{\cancel{x}}\right)\cancel{2}z \Rightarrow x^2 = z^2.$ Since x & $y \geq 0$ we get $x = z.$

$(4) \Rightarrow x^2+x^2+x^2 = 1 \Rightarrow 3x^2 = 1 \Rightarrow x = \frac{1}{\sqrt{3}}.$

$\therefore$ $\boxed{(x,y,z) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \text{ \& } V = \frac{8}{3\sqrt{3}} \quad \text{Side lengths} = \frac{2}{\sqrt{3}}}$