HOMEWORK 7: 16.1:12, 16.2: 17, 16.3:14, 20
16.4:30

16.1:12 If $R = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 5,\ 0 \le y \le 3\}$ then

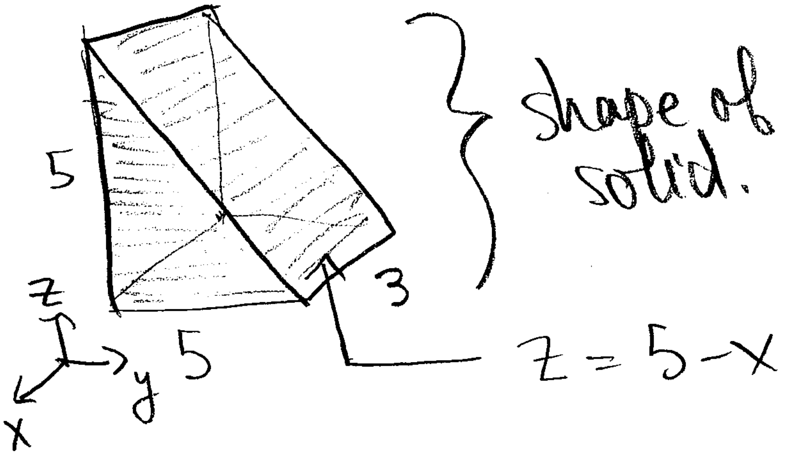
$$\iint_R (5-x)\,dA = \int_0^5 \int_0^3 (5-x)\,dy\,dx$$

$$= \int_0^5 \Big((5-x)3 \Big)\,dx$$

$$= 15x - \frac{3x^2}{2}\Big|_0^5$$

$$= 15 \cdot 5 - \frac{3 \cdot 25}{2}$$

$$= \frac{150 - 75}{2} = \frac{75}{2}.$$



Area of $_{5}\triangle_{5} = \frac{25}{2}$

$(\text{Area of } _{5}\triangle_{5}) \cdot (3) = \text{vol}$
$= \frac{75}{2}$

16.2:17 $R = \{(x,y): 0 \le x \le 1, -3 \le y \le 3\}$

$$\iint_R \frac{xy^2}{x^2+1}\,dA = \int_0^1 \int_{-3}^{3} \frac{xy^2}{x^2+1}\,dy\,dx$$

$$= \int_0^1 \frac{x}{x^2+1}\left\{\int_{-3}^{3} y^2\,dy\right\}dx$$

$$= \int_0^1 \frac{x}{x^2+1}\left\{\frac{y^3}{3}\Big|_{-3}^{3}\right\}dx$$

$$= \int_0^1 \frac{x}{x^2+1}\left\{\frac{27}{3}+\frac{27}{3}\right\}dx$$

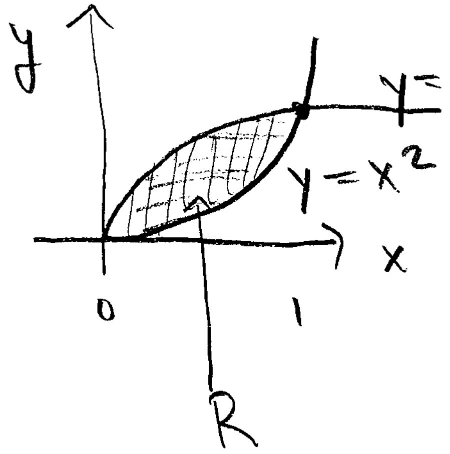
$$= 18\int_0^1 \frac{d}{dx}\left[\frac{1}{2}\ln(x^2+1)\right]dx$$

$$= 9\left(\ln(x^2+1)\Big|_0^1\right)$$

$$= 9\left(\ln(2)\right)$$

$$= 9\ln(2).//$$

16.3:14 Let R be the region in the xy-plane bounded by $y=\sqrt{x}$, $y=x^2$.



$x\in[0,1]$

$y\in[x^2,\sqrt{x}]$.

$$\iint_R (x+y)\,dA = \int_0^1\int_{x^2}^{\sqrt{x}}(x+y)\,dy\,dx$$

$$=\int_0^1\left\{\int_{x^2}^{\sqrt{x}}(x+y)\,dy\right\}dx$$

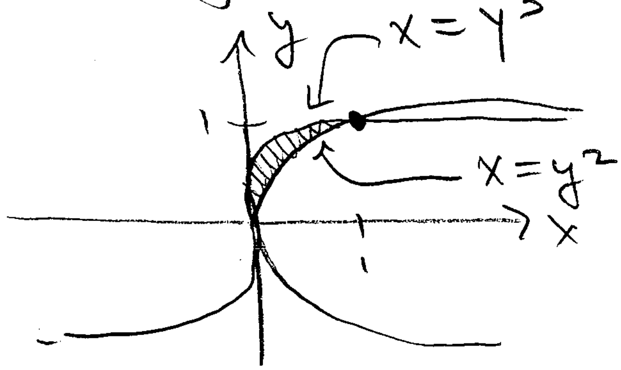
$$=\int_0^1\left(x(\sqrt{x}-x^2)+\frac{1}{2}\left((\sqrt{x})^2-(x^2)^2\right)\right)dx$$

$$=\int_0^1\left(-x^3+x\sqrt{x}-\frac{x^4}{2}+\frac{x}{2}\right)dx$$

$$=-\frac{x^4}{4}+\frac{x^{5/2}}{5/2}-\frac{x^5}{2\cdot 5}+\frac{x^2}{2\cdot 2}\Big|_0^1$$

$$=-\cancel{\frac{1}{4}}+\frac{2}{5}-\frac{1}{10}+\cancel{\frac{1}{4}}=\frac{3}{10}.//$$

16.3:20 Find the volume of region under $z = 2x + y^2$ over region in xy-plane bounded by $x = y^2$, $x = y^3$.



$y \in [0,1]$
$x \in [y^3, y^2]$

$$\iint_R (2x+y^2)\,dA = \int_0^1 \int_{y^3}^{y^2} (2x+y^2)\,dx\,dy$$

$$= \int_0^1 \left(x^2 + y^2 x \Big|_{y^3}^{y^2} \right) dy$$

$$= \int_0^1 (y^4 + y^4 - y^6 - y^5)\,dy$$

$$= \frac{2y^5}{5} - \frac{y^7}{7} - \frac{y^6}{6} \Big|_0^1$$

$$= \frac{2}{5} - \frac{1}{7} - \frac{1}{6}$$

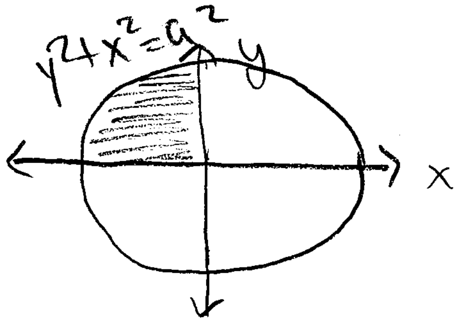
$$= \frac{19}{210}$$

$\boxed{16.4:30}$

$$\int_0^a \int_{-\sqrt{a^2-y^2}}^{0} x^2 y \, dx \, dy, \qquad y \in [0,a] \quad x \in [-\sqrt{a^2-y^2}, 0]$$

$y=0$ to $y=a$

$x=-\sqrt{a^2-y^2}$ to $x=0$ $\Big\} \rightsquigarrow$



IN POLAR: $\theta \in [\frac{\pi}{2}, \pi]$, $r \in [0,a]$.

$$\therefore \int_0^a \int_{-\sqrt{a^2-y^2}}^{0} x^2 y \, dx dy = \iint_R x^2 y \, dA$$

$$= \int_0^a \int_{\pi/2}^{\pi} (r\cos\theta)^2 (r\sin\theta) r \, d\theta \, dr$$

$$= \int_0^a \int_{\pi/2}^{\pi} r^4 ((\cos\theta)^2 \sin\theta) \, d\theta \, dr$$

$$= \int_0^a r^4 \left\{ \int_{\pi/2}^{\pi} \frac{d}{d\theta}\left[-\frac{\cos(\theta)^3}{3} \right] d\theta \right\} dr$$

$$= \int_0^a r^4 \left\{ \frac{1}{3} \right\} dr = \frac{1}{3} \left. \frac{r^5}{5} \right|_0^a = \frac{a^5}{15}. //$$