16.9: 2, 4, 10
17.1: 6, 26

HOMEWORK 9

16.9: 2

$x = uv$
$y = u/v$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} v & u \\ \frac{1}{v} & -\frac{u}{v^2} \end{vmatrix}$$

$$= -\frac{u}{v} - \frac{u}{v} = -2\frac{u}{v}. \quad //$$

16.9: 4

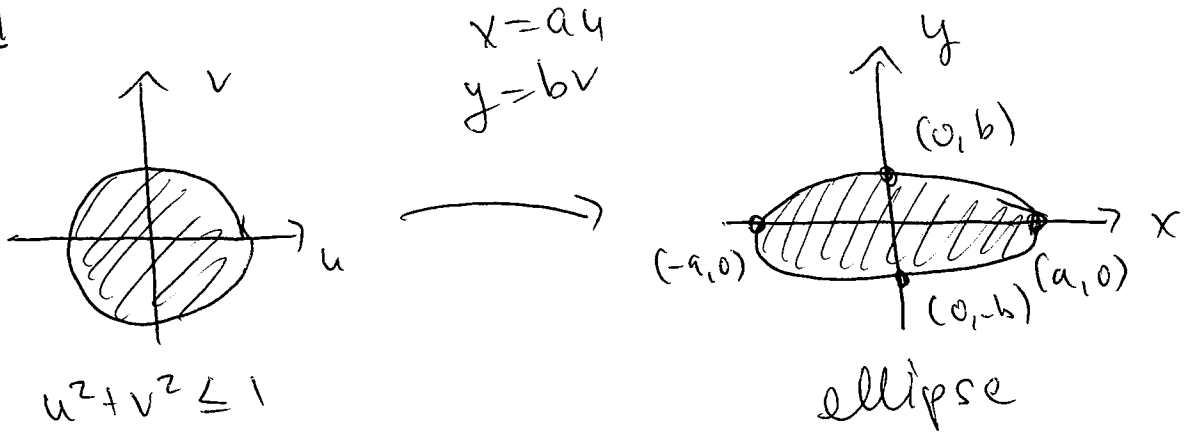
$x = e^{s+t}$
$y = e^{s-t}$

$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} e^{s+t} & e^{s+t} \\ e^{s-t} & -e^{s-t} \end{vmatrix}$$

$$= -e^{s+t}e^{s-t} - e^{s+t}e^{s-t}$$

$$= -2e^{2s}. \quad //$$

16.9:10



$u^2+v^2 \le 1$ ellipse

16.9:14 (Not Graded)

$$x = \sqrt{2}u - \sqrt{2/3}\,v$$
$$y = \sqrt{2}u - \sqrt{2/3}\,v$$

$$x^2 - xy + y^2 \le 2 \Rightarrow 2 \ge \left(\sqrt{2}u - \sqrt{2/3}\,v\right)^2 - \left(\sqrt{2}u - \sqrt{2/3}\,v\right)\left(\sqrt{2}u + \sqrt{2/3}\,v\right) + \left(\sqrt{2}u + \sqrt{2/3}\,v\right)$$

$$= 4u^2 + \frac{4}{3}v^2 - \left(2u^2 - \frac{2}{3}v^2\right)$$

$$= 2u^2 + \frac{6}{3}v^2 = 2u^2 + 2v^2$$

$\Rightarrow u^2 + v^2 \le 2$. ← new region

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \sqrt{2} & -\sqrt{2/3} \\ \sqrt{2} & \sqrt{2/3} \end{vmatrix} = \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}}.$$

$$\iint_R (x^2 - xy + y^2)\,dA = \iint_S 2(u^2+v^2)\cdot \frac{4}{\sqrt{3}}\,dA$$

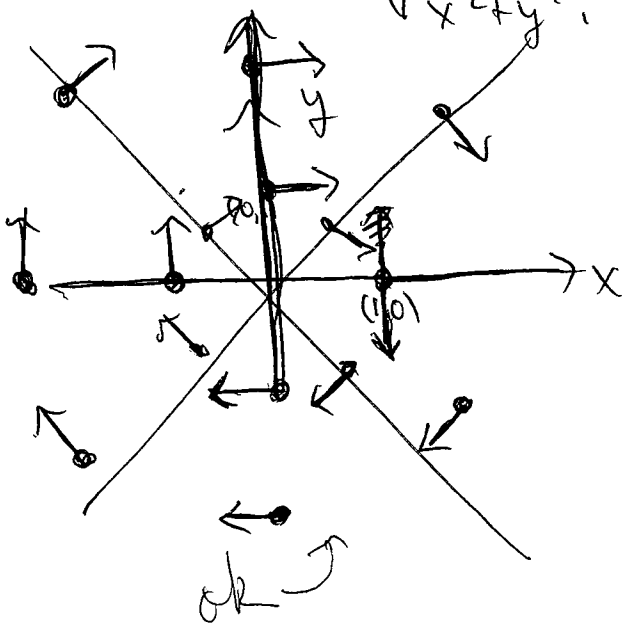
S: $u^2+v^2 \le 1$.

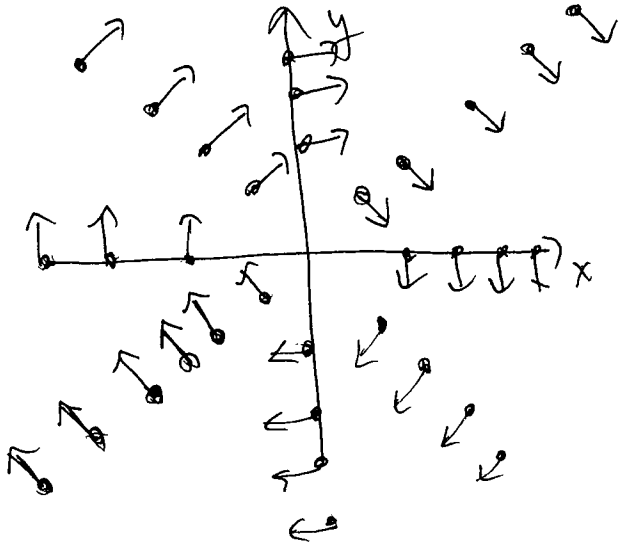
$$= \frac{8}{\sqrt{3}} \int_0^{2\pi}\int_0^1 r^3\,dr\,d\theta$$

$$= \frac{8}{\sqrt{3}} \cdot 2\pi \left(\frac{r^4}{4}\Big|_0^1 \right)$$

$$= \frac{4\pi}{\sqrt{3}} \cdot \frac{1}{1}$$

17.1:6 Plot $F = \dfrac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$.





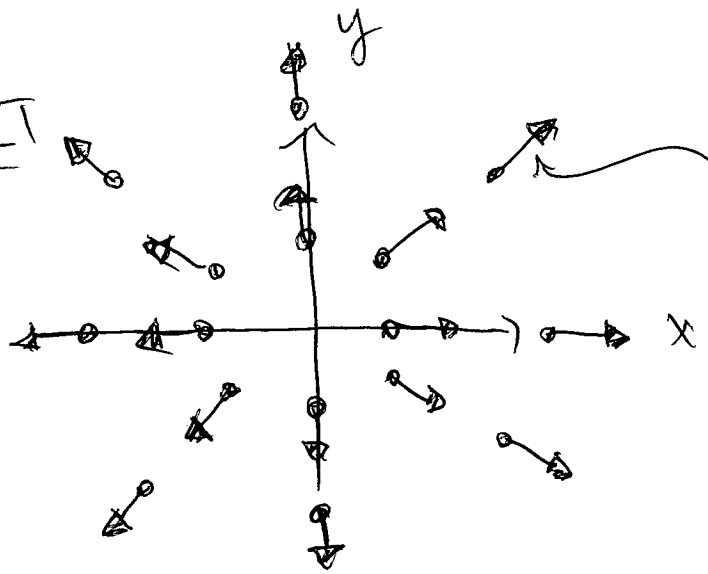
$$F(-1,1) = \frac{1\hat{i} - (-1)\hat{j}}{\sqrt{2}} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

17.1: 26 $f(x,y) = \sqrt{x^2+y^2}$

$$\nabla f = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right) = \underbrace{\frac{x}{\sqrt{x^2+y^2}}\,\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\,\hat{j}}$$

GRADIENT VECTOR FIELD OF f.

PLOT



all of these are unit vectors pointing in the same direction where the point is located.