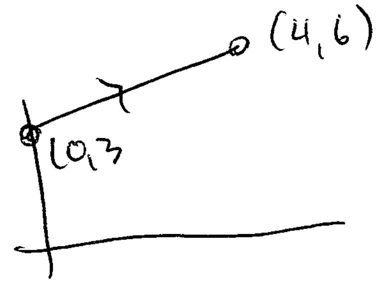
HOMEWORK 10

17.2: 4, 20
17.3: 16
17.4: 12
17.5: 14

17.2: 4

$$\int_C x\sin(y)\,ds$$



$r(t) = (0,3)(1-t) + t(4,6) \qquad t \in [0,1]$

$= (4t, 3+3t).$

$r'(t) = (4,3),$

$|r'(t)| = 5.$

$$\int_C x\sin(y)\,ds = \int_0^1 4t\sin(3+3t)(5\,dt)$$

$$= 20\left\{ t\,\frac{\cos(3+3t)}{-3}\Big|_0^1 - \int_0^1 \frac{\cos(3+2t)}{-3}\,dt \right\}$$

$$= \frac{20}{3}\left(-\cos(6) + \int_0^1 \frac{d}{dt}\left(\frac{\sin(3+3t)}{3}\right)dt\right)$$

$$= \frac{20}{3}\left(-\cos(6) - \frac{1}{3}\sin(3) + \frac{1}{3}\sin(6)\right)$$

17.2?

20. $\cancel{F(x,y,z) = \sin(x)\hat{i} + \cos(y)\hat{j} +}$

$$\begin{cases} F(x,y,z) = (x+y)\hat{i} + (y-z)\hat{j} + z^2\hat{k} \\ r(t) = t^2\hat{i} + t^3\hat{j} + t^2\hat{k}, \quad t \in [0,1] \end{cases}$$

$$r'(t) = 2t\hat{i} + 3t^2\hat{j} + 2t\hat{k}$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 F(r(t)) \cdot r'(t)\,dt$$

$$= \int_0^1 \left[(t^2+t^3)\hat{i} + (t^3-t^2)\hat{j} + (t^2)^2\hat{k}\right] \cdot \left[2t\hat{i} + 3t^2\hat{j} + 2t\hat{k}\right] dt$$

$$= \int_0^1 \left[2t(t^2+t^3) + 3t^2(t^3-t^2) + 2t(t^4)\right] dt$$

$$= \int_0^1 \left[2t^3 + 2t^4 + 3t^5 - 3t^4 + \cancel{5t}\, 2t^5\right] dt$$

$$= \int_0^1 \left[2t^3 - t^4 + 5t^5\right] dt$$

$$= \cancel{\int_0^1} 2\frac{t^4}{4}\Big|_0^1 - \frac{t^5}{5}\Big|_0^1 + 5\frac{t^6}{6}\Big|_0^1$$

$$= \frac{2}{4} - \frac{1}{5} + \frac{5}{6} = \frac{15}{30} - \frac{6}{30} + \frac{25}{30} = \frac{34}{30} = \frac{17}{15}. \;//$$

17.3:16 : (a) find potential
(b) Evaluate $\int_C F \cdot dr$

$$F = (2xz + y^2)\hat{\imath} + 2xy\hat{\jmath} + (x^2 + 3z^2)\hat{k}$$

C: $x(t) = t^2$, $y(t) = t+1$, $z(t) = 2t-1$, $t \in [0,1]$

If $\nabla u = F$,

$$\begin{cases} \dfrac{\partial u}{\partial x} = 2xz + y^2 \\[2mm] \dfrac{\partial u}{\partial y} = 2xy \\[2mm] \dfrac{\partial u}{\partial z} = x^2 + 3z^2 \end{cases} \Rightarrow \overbrace{\begin{array}{l} (1)\ u(x,y,z) = x^2 z + y^2 x + C_1(y,z) \\ (2)\ u(x,y,z) = xy^2 + C_2(x,z) \\ (3)\ u(x,y,z) = x^2 z + z^3 + C_3(y,x) \end{array}}^{\star}$$

using (1) & (2) we have

$$x^2 z + \cancel{y^2 x} + C_1(y,z) = \cancel{xy^2} + C_2(x,z)$$

Apply $\frac{\partial}{\partial y}$ to both sides we get

$$\frac{\partial}{\partial y}\left[C_1(y,z)\right] = 0$$

$\Rightarrow C_1(y,z)$ actualy only depends on z; $C_1(y,z) = C_1(\cancel{y},z)$

using (1) & (3)

$$\cancel{x^2 z} + y^2 x + C_1(z) = \cancel{x^2 z} + z^3 + C_3(y,x)$$

$\Rightarrow C_1(z) = z^3$

$C_3(y,\cancel{x}) = y^2 x$

$\therefore \boxed{u(x,y,z) = x^2 z + y^2 x + z^3.}$

NOTE: $\nabla u = (2xz + y^2, 2yx, x^2 + 3z^2) = F.$

Remark: It is probably best to just look at the eqns (★) then guess & check.

The end points of C are $r(0) = (0,1,-1)$ and $r(1) = (1,2,1)$. So by the fundamental theorem of line integrals,

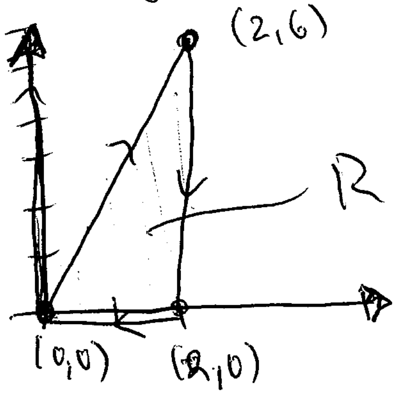
$$\int_C F \cdot dr = u(1,2,1) - u(0,1,-1)$$

$$= \left[(1)^2(1) + (2)^2(1) + (1)^3\right] - \left[(0)^2(-1) + (1)^2(0) + (-1)^3\right]$$

$$= 6 + 1$$

$$= 7. \;//$$

17.4: 12 $\quad F(x,y) = \left(y^2\cos(x),\ x^2 + 2y\sin(x)\right)$



Green's Thm now applies

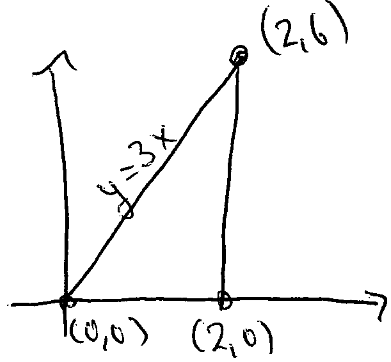
$$\int_C \vec{F} \cdot d\vec{r} = -\int_{-C} \vec{F} \cdot d\vec{r} = -\left(\int_{-C} \vec{F} \cdot d\vec{r}\right)$$

$$= -\left[\int_{-C} y^2\cos(x)\,dx + (x^2 + 2y\sin(x))\,dy\right]$$

$$= -\iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dx\,dy$$

$$= -\iint_R \left[(2x + 2y\cos(x)) - (2y\cos(x)) \right] dx\,dy$$

$$= -\iint_R 2x \, dx\,dy,$$



$y \in [0, 3x]$
$x \in [0, 2]$

$$= -\int_0^2 \int_0^{3x} 2x \, dy\, dx$$

$$= -\int_0^2 6x^2 \, dx$$

$$= -\left[6 \frac{x^3}{3} \Big|_0^2 \right]$$

$$= -[2 \cdot 8]$$

$$= -16. //$$

17.5 : 14 ~~F xyz² î + x²yz² ĵ + x²y²z k̂ is conservative since u(x,y,z) =~~

$$F = xyz^2\hat{\imath} + x^2yz^2\hat{\jmath} + x^2y^2z\hat{k},$$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ xyz^2 & x^2yz^2 & x^2y^2z \end{vmatrix}$$

$$= i\left(2x^2yz - 2x^2yz\right) \cancel{+} - j\left(2xy^2z - 2xyz\right)$$

$$+ k\left(2xyz^2 - xz^2\right)$$

$$\neq 0$$

$\therefore$ the vector field is not conservative.//

(THM 4 of this chapter)