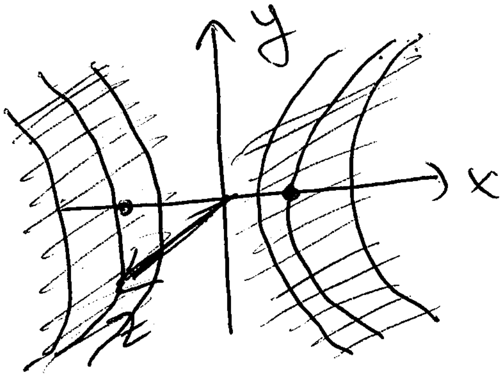
HW03:

13.6: 8, 18, ~~44~~, 41

~~13~~, 14.1: 8, ~~16~~ 42

13.6:8 Describe $x^2 - y^2 = 1$ as a surface.

soln: $\{(x,y,z) \in \mathbb{R}^3 \mid x^2 - y^2 = 1\}$ is a hyperbolic cylinder



13.6:18 Use traces to sketch & identify the surface

$$4x^2 - 16y^2 + z^2 = 16$$

soln:

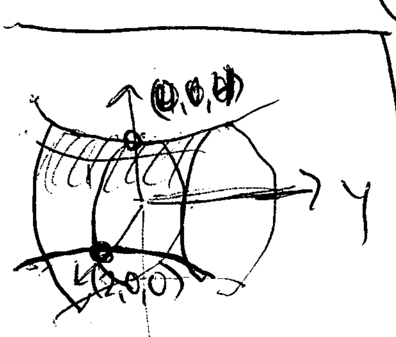
$$\frac{x^2}{4} - y^2 + \frac{z^2}{16} = 1$$

hyperboloid of one sheet symmetric about the y-axis.

$$\Longleftrightarrow \quad \left(\frac{x}{2}\right)^2 - y^2 + \left(\frac{z}{4}\right)^2 = 1$$

stretched in z direction by a factor of 4

stretched in x direction by a factor of 2.



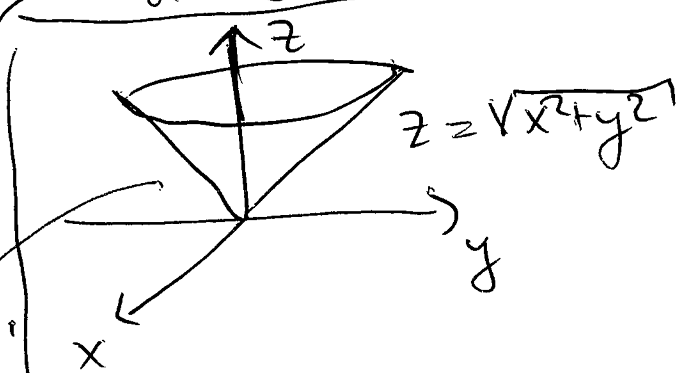
~~13.6:40~~

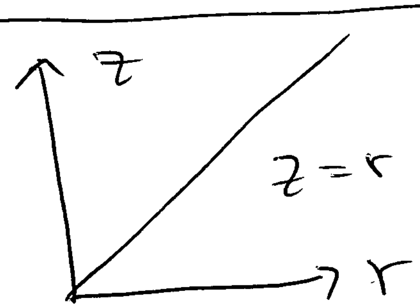
13.6:41 sketch the region bounded by the surfaces $z=\sqrt{x^2+y^2}$ & $x^2+y^2=1$ for $z\in[1,2]$.

soln: • $z=\sqrt{x^2+y^2}$ ← a cone

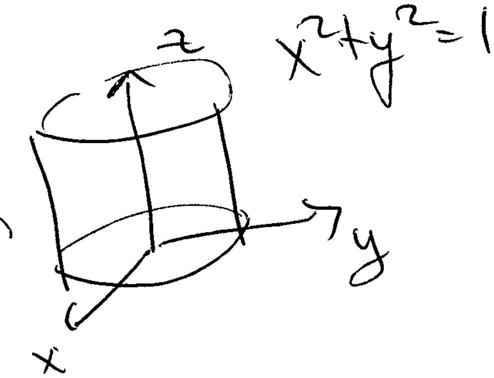
$=\sqrt{r^2}$

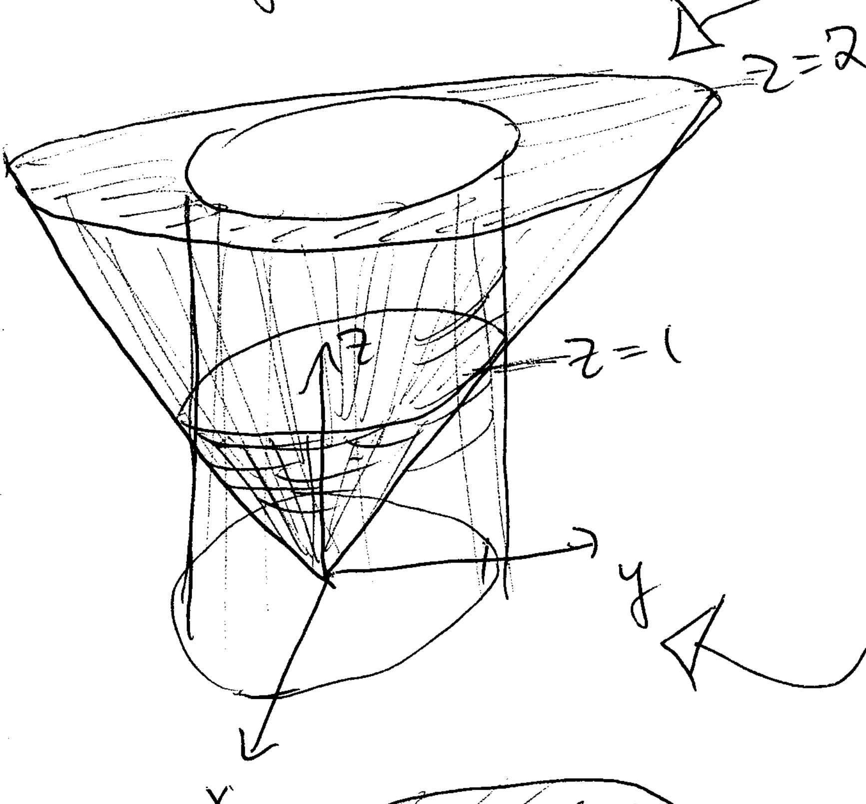
$=r$

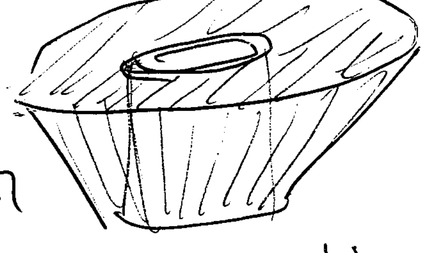




• $x^2+y^2=1$ is a cylinder.





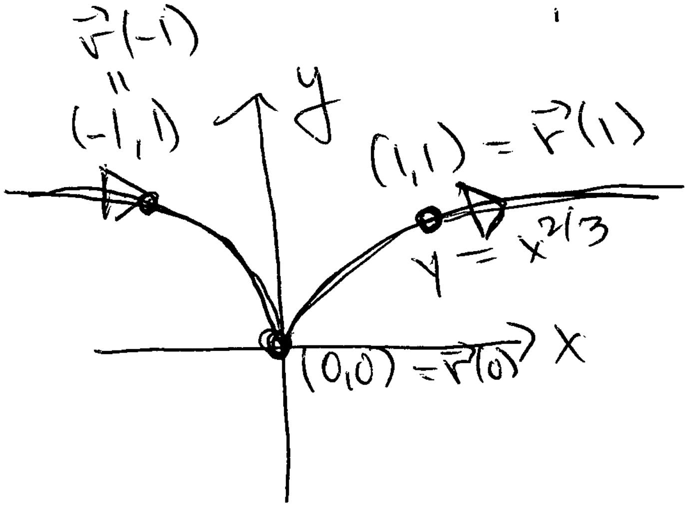


region bounded by $z=\sqrt{x^2+y^2}$ & $x^2+y^2=1$ for $z\in[1,2]$.

14.1:8 sketch the curve parametrized by

$$\vec{r}(t) = (t^3, t^2).$$

soln: $x = t^3$, $y = t^2$. $\Rightarrow$ $y = x^{2/3}$ or $y^3 = x^2$.



14.1:42 $\vec{r}_1(t) = (t, t^2, t^3)$

$\vec{r}_2(t) = (1+2t, 1+6t, 1+14t)$

- do particles ~~intersect~~ paths intersect?
- do the particles collide?

soln: $r_1(t) = r_2(s)$

$$\Rightarrow \begin{cases} t = 1+2s \\ t^2 = 1+6s \\ t^3 = 1+14s \end{cases} \Rightarrow (1+2s)^2 = 1+6s$$

$\Rightarrow 4s^2 + 4s + 1 - (1+6s) = 0$

$\Rightarrow 4s^2 - 2s = 0 \Rightarrow 2s(2s-1) = 0$

$\Rightarrow s = 0$ or $s = 1/2$

when $s=0$, $t=1$; when $s=\frac{1}{2}$, $t=2$,

- $r_1(1)=r_2(0)$? $r_1(1)=(1,1,1)$
  $r_2(0)=(1,1,1)$ so ok. ✓

paths intersect at the point $(1,1,1)$.

~~$r_1(3)=r_2(1)$? $r_1(3)=(3,9,27)$~~
~~$r_2(1)=(3,7,15)$~~

- $r_1(2)=r_2(1/2)$? $r_1(2)=(2,4,8)$ so ok ✓.
  $r_2(1/2)=(2,4,8)$

paths intersect at the point $(2,4,8)$.

☆ they never collide since $s\neq t$ at the places of intersection.