HOMEWORK 04

14.2: 12, 26, ~~30~~ 32
14.4: ~~24~~ 12, 40

14.2:12 Find the derivative of

$$\vec{r}(t) = \sin^{-1}t\,\hat{\imath} + \sqrt{1-t^2}\,\hat{\jmath} + \hat{k}$$

soln: $y = \sin^{-1}t \Rightarrow \sin y = t$

$$\Rightarrow \cos(y)\,y' = 1$$

$$\Rightarrow y' = 1/\cos(y) = 1/\sqrt{1-\sin(y)^2}$$

$$= 1/\sqrt{1-t^2}$$

$$\vec{r}\,'(t) = \frac{1}{\sqrt{1-t^2}}\,\hat{\imath} + \frac{1}{2}(1-t^2)^{-1/2}(-2t)\,\hat{\jmath} + 0\,\hat{k}$$

$$= \frac{1}{\sqrt{1-t^2}}\,\hat{\imath} + \frac{-t}{\sqrt{1-t^2}}\,\hat{\jmath}.$$

14.2:26: Find parametric eqns for the line tangent to the curve at the specified point.

$$x = \ln t$$

$$y = 2\sqrt{t}$$

$$z = t^2$$

$P = (0, 2, 1)$.

14.2:26 cont...

Soln

$$\begin{cases} x'(t) = 1/t \\ y'(t) = t^{-1/2} \\ z'(t) = 2t \end{cases},$$

$$(\ln(t), 2\sqrt{t}, t^2) = (0,2,1)$$

when $\begin{cases} \ln(t) = 0 \\ 2\sqrt{t} = 2 \\ t^2 = 1 \end{cases}$

$\Rightarrow t = 1.$

FORMULA FOR TANGENT LINE:

$$\vec{l}(t) = \vec{r}(t_0) + \vec{r}\,'(t_0)(t-t_0)$$

$t_0 = 1$

$\vec{r}(t_0) = (0,2,1)$

$\vec{r}\,'(t_0) = (1,1,2)$

$$\Rightarrow \vec{l}(t) = (0,2,1) + (1,1,2)(t-1)$$

$$= (t-1, t+1, 2t)$$

components for the tangent line.

<u>14.2:32</u>: At what points do the curves parametrized by

$$\vec{r}_1(t) = (t, 1-t, 3+t^2)$$
$$\vec{r}_2(t) = (3-t, t-2, t^2)$$

intersect? What is the angle of intersection?

<u>soln</u>. $\vec{r}_1(t) = \vec{r}_2(s) \iff \begin{cases} t = 3-s \\ 1-t = s-2 \\ 3+t^2 = s^2 \end{cases}$

$\Rightarrow 1-(3-s) = s-2 \Rightarrow s-2 = s-2$, so the first & second equations are ~~redundant~~, linearly dependent & both are ~~are always~~ going to be satisfied, provided one of them is satisfied. We still need to check consistency with the 3rd eqn.

$$3+(3-s)^2 = s^2 \iff 3+9-6s+s^2 = s^2$$
$$\iff 12 = 6s$$
$$\iff s = 2.$$

When $s=2$ we have $t = 3-(2) = 1$. So the point of intersection is at $\vec{r}_1(1) = \vec{r}_2(2) = (1,0,4)$.

The angle of intersection is given by

$$\theta = \cos^{-1}\left(\frac{\vec{r}_1{}'(1) \cdot \vec{r}_2{}'(2)}{|\vec{r}_1{}'(1)|\,|\vec{r}_2{}'(2)|} \right).$$

$\vec{r}_1{}'(t) = (1,-1,2t) \Rightarrow \vec{r}_1{}'(1) = (1,-1,2).$

$\vec{r}_2{}'(t) = (-1,1,2t) \Rightarrow \vec{r}_2{}'(2) = (-1,1,4).$

$|\vec{r}_1{}'(1)| = \sqrt{1+1+4} = \sqrt{6}$

$|\vec{r}_2{}'(2)| = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}.$

$\vec{r}_1{}'(1) \cdot \vec{r}_2{}'(2) = \cancel{8}.6$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{6}{\sqrt{6}\,3\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{6}}{3\sqrt{3}\sqrt{2}}\right). = \cos^{-1}\left(\frac{1}{3}\right)$$

<u>14.4; 12</u>: Find velocity, accel & speed of particle with position function

$$\vec{r}(t) = t^2\hat{\imath} + \ln t\,\hat{\jmath} + t\,\hat{k},$$

<u>soln</u>.

$\vec{r}\,'(t) = 2t\hat{\imath} + \frac{1}{t}\hat{\jmath} + \hat{k}$ velocity.

$|\vec{r}\,'(t)| = \sqrt{4t^2 + 1/t^2 + 1}$ speed.

$\vec{r}\,''(t) = 2\hat{\imath} + \frac{-1}{t^2}\hat{\jmath}$ accel.

14.4.40

$$\underbrace{\vec{l}(t) := m\vec{r}(t)\times\vec{v}(t)}_{\text{angular momentum vector}}$$

$$\underbrace{\vec{\tau}(t) := m\vec{r}(t)\times\vec{a}(t)}_{\text{torque vector.}}$$

1) Show $\vec{l}\,'(t) = \vec{\tau}(t)$.

pf.

$$\frac{d}{dt}\left[\vec{l}(t)\right] = \frac{d}{dt}\left[m\vec{r}(t)\times\vec{v}(t)\right]$$

Product Rule For Cross Products.

$$\left\{\begin{aligned} &= \frac{d}{dt}\left[m\vec{r}(t)\right]\times\vec{v}(t) \\ &\quad + m\vec{r}(t)\times\frac{d}{dt}\left[\vec{v}(t)\right]\end{aligned}\right.$$

$$= m\frac{d}{dt}\left[\vec{r}(t)\right]\times\vec{v}(t) + m\vec{r}(t)\times\frac{d}{dt}\left[\vec{v}(t)\right]$$

$$= m\left(\cancelto{0}{\vec{v}(t)\times\vec{v}(t)}\right) + m\vec{r}(t)\times\vec{a}(t)$$

$$= m\vec{r}(t)\times\vec{a}(t).\ //$$

2) If $\vec{\tau}(t) = \vec{0}$ then $\vec{l}(t)$ is constant.

pf. Let $\vec{l}(t) = l_x(t)\hat{\imath} + l_y(t)\hat{\jmath} + l_z(t)\hat{k}$.

$$\vec{0} = \vec{\tau}(t) = \frac{d}{dt}\left[\vec{l}(t)\right] = l_x'(t)\hat{\imath} + l_y'(t)\hat{\jmath} + l_z'(t)\hat{k}.$$

$\Rightarrow$ $l_x'(t) = 0$
$l_y'(t) = 0$ $\Rightarrow$ l_x, l_y & l_z are all constant.
$l_z'(t) = 0$

(the derivative of a function is zero for all inputs iff that function is a constant)

$\Rightarrow$ $\vec{l}(t)$ is constant since all of its components are. //