HOMEWORK 9

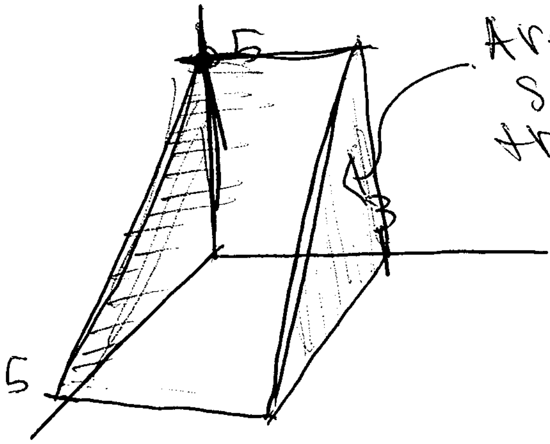
16.1 : 12
16.2 ; 8
16.3 ; 14, 20
16.4 : 30

<u>16.1:12</u> Evaluate the integral by identifying it with a vol of a solid

8 &

$$\iint_R (5-x)\,dA, \quad R = \{(x,y) \mid 0 \le x \le 5,\ 0 \le y \le 3\} = [0,5]\times[0,3]$$

<u>Soln</u>. This is the region below the plane $z = 5-x$ and above the rectangle R.



Area of side triangle $= \frac{1}{2}$ base $\cdot$ height

$$= \frac{1}{2}(5)\cdot(5)$$

$$= \frac{25}{2}.$$

Area of solid $=$ (Area of side Δ) $\cdot\ 3$

$$= \frac{75}{2}$$

$$\therefore \boxed{\iint_R (5-x)\,dA = \frac{75}{2}}$$

//

16.2:8

$$\int_0^1\int_1^2 \frac{xe^x}{y}\,dy\,dx = \left(\int_1^2 \frac{1}{y}\,dy\right)\left(\int_0^1 xe^x\,dx\right)$$

$$= \left(\ln(y)\Big|_{y=1}^{y=2}\right)\left(xe^x\Big|_{x=0}^{x=1} - \int_0^1 e^x\,dx\right)$$

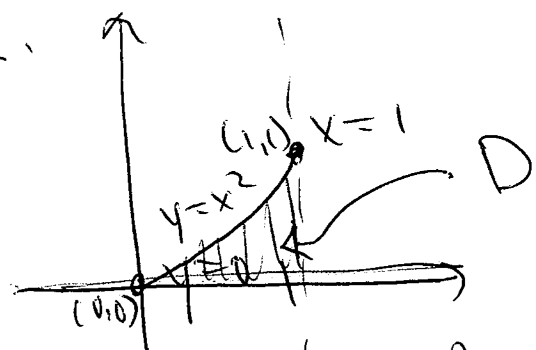
$$= \ln(2)\left(e - (e-1)\right)$$

$$= \ln(2).$$

16.3:14

$$\iint_D (x+y)\,dA \quad , D \text{ bounded by } \begin{array}{l} y=0 \\ y=x^2 \\ x=1 \end{array}$$

soln.



$y \in [0, x^2]$
$x \in [0, 1]$

$$\iint_D (x+y)\,dA = \int_0^1\int_0^{x^2} (x+y)\,dy\,dx$$

$$= \int_0^1 x^3 + \frac{y^2}{2}\Big|_{y=0}^{y=x^2}\,dx$$

$$= \int_0^1 x^3 + \frac{x^4}{2}\,dx$$

$$= \frac{x^4}{4} - \frac{x^5}{10}\Big|_{x=0}^{x=1} = \frac{1}{4} - \frac{1}{10} = \frac{10}{40} - \frac{4}{40}$$

$$= \frac{6}{40} = \frac{3}{20}.$$

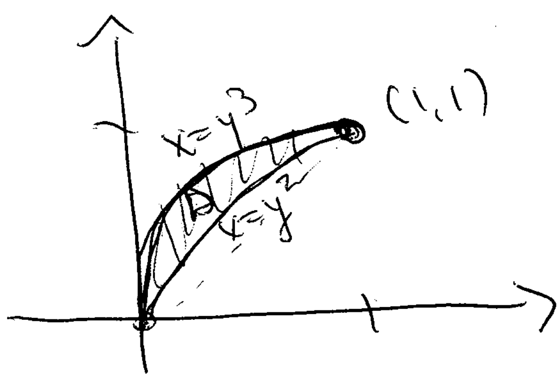
[I actually copied the problem down wrong the answer to the problem with the correct bounds is 3/10]

<u>16.3:20</u> Find the volume of the solid under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ & $x = y^3$.

<u>Soln.</u>

$$\text{Vol} = \iint_D (2x+y^2)\,dA$$

when D is the region



$x \in [y^3, y^2]$
$y \in [0,1]$

$$\Rightarrow \iint_D (2x+y^2)\,dA = \int_0^1 \int_{y^3}^{y^2} (2x+y^2)\,dx\,dy$$

$$= \int_0^1 x^2\Big|_{x=y^3}^{x=y^2} + y^2(y^2-y^3)\,dy$$

$$= \int_0^1 (y^4 - y^6) + y^4 - y^{\cancel{4}\,5}\,dy$$

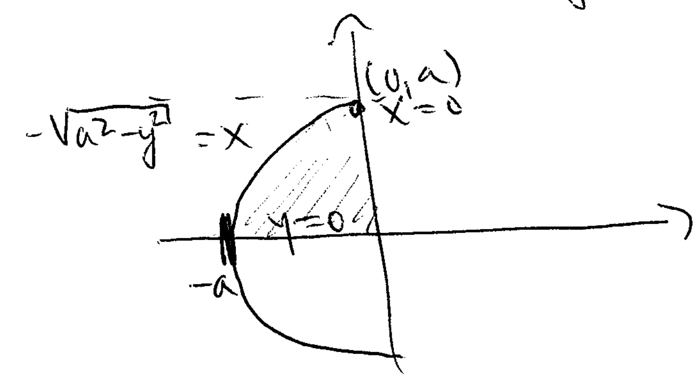
$$= \cancel{2}\int_0^1 (2y^4 - y^6 - y^5)\,dy = \cancel{2}\left(\frac{2}{5} - \frac{1}{7} - \frac{1}{6}\right) = \frac{19}{200}$$

$$\cancel{= 2\left(\frac{7}{35} - \frac{5}{35}\right) = \frac{4}{35}} \;//$$

<u>16.4:30</u> Evaluate the integral by converting to polar coordinates,

$$\int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y \,dx\,dy$$

<u>soln</u>. Draw the region



$\theta \in \left[\frac{\pi}{2}, \pi\right]$
$r \in [0, a]$

$$\int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y\,dx\,dy = \int_{\pi/2}^{\pi} \int_0^a (r\cos(\theta))^2 (r\sin(\theta))\, r\,dr\,d\theta$$

$$= \left(\int_{\pi/2}^{\pi} \cos(\theta)^2 \sin(\theta)\,d\theta\right)\left(\int_0^a r^4\,dr\right)$$

$u = \cos(\theta)$
$du = -\sin(\theta)\,d\theta$

$$= \left(\int_0^{-1} -u^2\,du\right)\left(\frac{a^5}{5}\right) = \frac{a^5}{5}\int_{-1}^0 u^2\,du = \frac{a^5}{5}\left(\frac{1}{3}\right) = \frac{a^5}{15} \;//$$