16.5: 2
16.6: 14, 16, 22
16.7: 26

HOMEWORK 10

16.5:2 $R = \{(x,y) \mid x^2+y^2 \le 4\}$

$$\iint_R \sigma(x,y)\, dA = \iint_R (x+y+x^2+y^2)\, dA$$

$$= \int_0^{2\pi}\int_0^2 (r\cos\theta + r\sin\theta + r^2)\, r\, dr\, d\theta$$

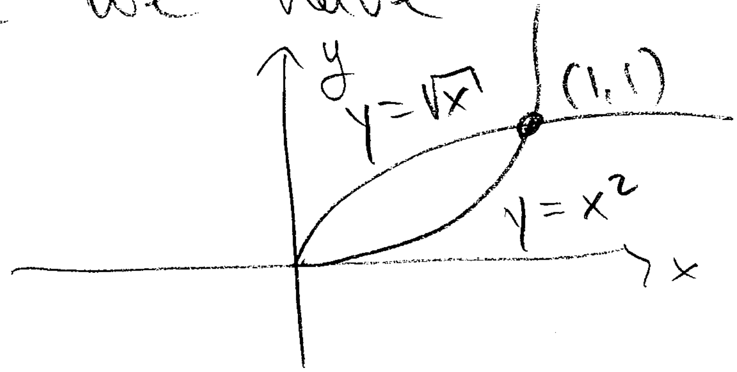
$$= \cancel{\int_0^{2\pi}\int_0^2 r^2\cos\theta\, dr\, d\theta}^{\,0} + \cancel{\int_0^{2\pi}\int_0^2 r^2\sin\theta\, dr\, d\theta}^{\,0}$$

$$+ \int_0^{2\pi}\int_0^2 r^3\, dr\, d\theta$$

$$= 2\pi \left.\frac{r^4}{4}\right|_{0=r}^{2=r} = 8\pi.$$

16.6:14: Compute $\iiint_E xy\, dV$ where E is the region bounded by the parabolic cylinders $x^2=y$ & $y^2=x$ & the planes $z=0$ & $z=x+y$.

soln. Let $z \in [0, x+y]$ so that in the xy-plane we have 2



$y \in [x^2, \sqrt{x}]$
$x \in [0,1]$.

So,

$$\iiint_E xy\,dV = \int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} xy \,dz\,dy\,dx$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} xy(x+y)\,dy\,dx$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} (x^2 y + xy^2)\,dy\,dx$$

$$= \int_0^1 \left[x^2 \frac{y^2}{2} + x \frac{y^3}{3} \Big|_{y=x^2}^{y=\sqrt{x}} \right] dx$$

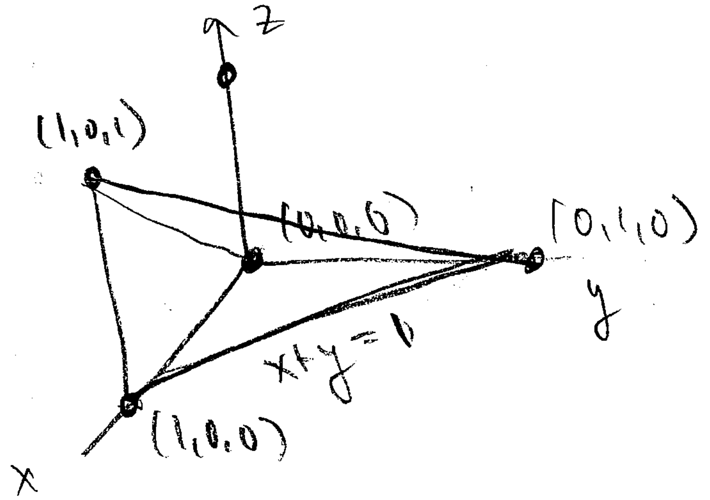
$$= \int_0^1 \left[x^2 \frac{x}{2} + x\frac{x^{3/2}}{3} - \left(x^2 \frac{x^4}{2} + x \frac{x^6}{3} \right) \right] dx$$

$$= \int_0^1 \left(\frac{x^3}{2} + \frac{x^{5/2}}{3} - \frac{x^6}{2} + \frac{x^7}{3} \right) dx$$

$$= \frac{1}{8} + \frac{2}{27} - \frac{1}{14} - \frac{1}{24}$$

16.6:16

$\iiint_T xyz\,dV$ where T is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ & $(1,0,1)$.



Find eqn for the top plane:

$$A + C + D = 0$$
$$B + D = 0$$
$$D = 0$$
$$\Rightarrow A = -C,\ B = 0,\ D = 0.$$

$$Ax - Az = 0 \Rightarrow x = z.$$

$$\begin{cases} z \in [0, x] \\ y \in [0, 1-x] \\ x \in [0,1] \end{cases}$$

$$\iiint_T xyz\,dV = \int_0^1 \int_0^{1-x} \int_0^x xyz\,dz\,dy\,dx$$

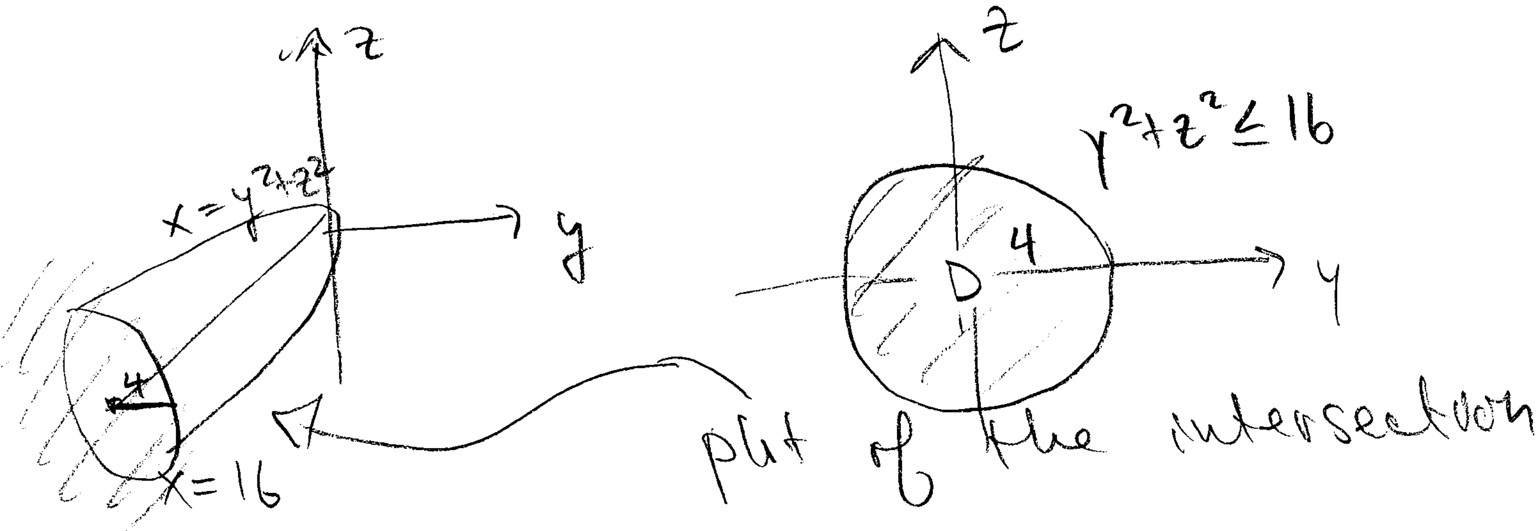
$$= \int_0^1 \int_0^{1-x} xy\frac{x^2}{2}\,dy\,dx$$

$$= \int_0^1 \frac{x^3}{2}\,\frac{(1-x)^2}{2}\,dy\,dx$$

$$= \frac{1}{4}\int_0^1 (x^3 - 2x^5 + x^7)\,dx$$

$$= \frac{1}{4}\left(\frac{1}{4} - \frac{2}{6} + \frac{1}{8}\right).$$

<u>16.6:22</u> Find volume of solid enclosed by paraboloid $x = y^2+z^2$ and plane $x=16$.



$$\iiint_E dV = \iint_D \int_{y^2+z^2}^{16} dx\, dA$$

$$= \iint_D \left(16 - (y^2+z^2)\right) dA$$

$$= \int_0^{2\pi}\int_0^4 (16 - r^2)\, r\, dr\, d\theta$$

$$= 2\pi \left(\int_0^4 16r - r^3 \, dr \right)$$

$$= 2\pi \left(16 \frac{r^2}{2} - \frac{r^4}{4} \Big|_{r=0}^{r=4} \right)$$

$$= 2\pi \left(8(4)^2 - \frac{4^4}{4} \right)$$

$$= 2\pi \left(2 \cdot 4^3 - 4^3 \right) = 2\pi (4^3) = 128\pi.$$

<u>16.7:26</u>: Find the mass of the ball B given by $x^2+y^2+z^2 \leq a^2$ if the density at a point is proportional to the distance to the z-axis.

$$\rho(x,y,z) = K\sqrt{x^2+y^2}.$$

$$\text{MASS} = \iiint_B \rho(x,y,z) \, dV$$

spherical coord

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{a} K\sqrt{r^2 \sin(\phi)^2} \, r^2 \sin(\phi) \, dr \, d\phi \, d\theta$$

$$= K(2\pi) \left(\int_0^a r^3 \, dr \right) \left(\int_0^{\pi} \sin(\phi)^2 \, d\phi \right)$$

$$= K(2\pi)\left(\frac{a^4}{4}\right)(\pi) = \frac{a^4 \pi^2 K}{2}.$$