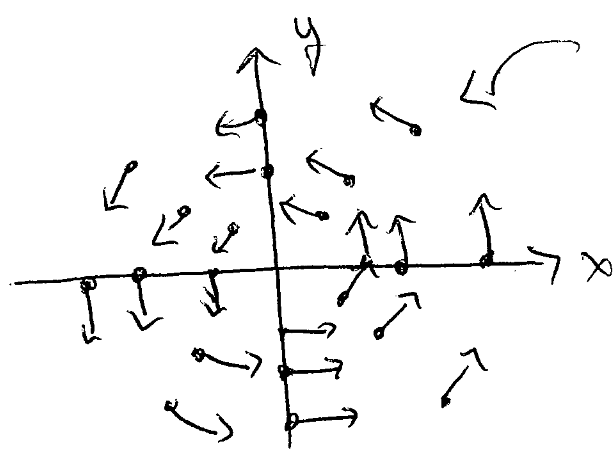
17.1: 6
17.2: 4, 20
17.3: 4, 26
17.4: 8, 14
17.5: 12

HOMEWORK 12 & 13

17.1:6: $\vec{F} = \dfrac{y\hat{i} - x\hat{j}}{\sqrt{x^2+y^2}}$



all unit vectors

17.2:4

$$\int_C x\sin(y)\,ds$$, C is line segment from (0,3) to (4,2).

soln: $\vec{r}(t) = \cancel{(0,3)t + (1-t)}$

$= (4,2)t + (0,3)(1-t), \quad t \in [0,1]$

$= (4t, 2t + 3 - 3t)$

$= (4t, -t+3)$

$$\int_C x \sin(y)\,ds = \int_0^1 4t \sin(-t+3)\sqrt{4^2+1}\,dt$$

$$= 4\sqrt{17}\int_0^1 \underbrace{\cancel{4}t}_{u}\ \underbrace{\sin(3-t)\,dt}_{dv}$$

$$= 4\sqrt{17}\left(t\cos(3-t)\Big|_{t=0}^{t=1} - \int_0^1 \cos(3-t)\,dt \right)$$

$$= 4\sqrt{17}\left(\cos(2) - \int_0^1 \frac{d}{dt}\left[-\sin(3-t)\right] dt \right)$$

$$= 4\sqrt{17}\left(\cos(2) - \left[-\sin(3-t)\Big|_{t=0}^{t=1} \right) \right.$$

$$= 4\sqrt{17}\left(\cos(2) - \left(-\sin(2) + \sin(3) \right) \right]$$

$$= 4\sqrt{17}\left(\cos(2) + \sin(2) - \sin(3) \right). //$$

<u>17.2.20</u> Evaluate

$$\int_C \vec{F}\cdot d\vec{r}$$

where $\vec{F} = (x+y)\hat{i} + (y-z)\hat{j} + z^2\hat{k}$ & $\vec{r}(t) = t^3\hat{i} - t^2\hat{j} + t\hat{k}$ where $0 \le t \le 1$.

Soln. $\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t))\cdot \vec{r}\,'(t)\,dt$

$$\cancel{= \int_0^1 \left((t^3-t^2)\hat{\imath} \right.}$$

$$= \int_0^1 \left((t^3-t^2)\hat{\imath} + (-t^2-t)\hat{\jmath} + t^2\hat{k} \right)\cdot\left(3t^2\hat{\imath} - 2t\hat{\jmath} + \hat{k}\right) dt$$

$$= \int_0^1 \left((3t^2)(t^3-t^2) + (-t^2-t)(-2t) + t^2 \right) dt$$

$$= \int_0^1 \left(3t^5 - 3t^4 + 2t^3 + 2t^2 + t^2 \right) dt$$

$$\cancel{= \int_0^1 \left(3t^5 \right.}$$

$$= \frac{3}{6} - \frac{3}{5} + \frac{2}{4} + \frac{2}{3} + \frac{1}{3}$$

$$= \frac{7}{5}.$$

17.3:4: Determine if $\vec{F}(x,y) = e^x\cos(y)\hat{\imath} + e^x\sin(y)\hat{\jmath}$ is conservative, if it is find a function f such that $\nabla f = \vec{F}$.

Soln: Since $\vec{F}$ is defined everywhere we know that $\vec{F}$ is conservative iff $\vec{F}$ has a potential, we can test for path indep:

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial}{\partial x}\left(e^x \sin(y)\right) - \frac{\partial}{\partial y}\left(e^x \cos(y)\right)$$

$$= e^x \sin(y) + e^x \sin(y)$$

$$\neq 0$$

So the vector field is not conservative.

17.3:26 ~~Let $\vec{F} = f(x,y)$~~ Let $\vec{F} = \nabla f$ where $f(x,y) = \sin(x-2y)$. Find curves C_1 & C_2 that are not closed and satisfy

a) $\int_{C_1} \vec{F} \cdot d\vec{r} = 0$

b) $\int_{C_2} \vec{F} \cdot d\vec{r} = 1.$

Soln: It is actually necessary that they not be closed so demanding that they be not closed is silly.

The problem is to find two points P_2 & P_1 such that $f(P_2) - f(P_1)$ is the desired difference. It doesn't

17.4:14 $\vec{F}(x,y) = (y - \ln(x^2+y^2))\hat{i} + (2\tan^{-1}(y/x))\hat{j}$ ⑥

C is the circle $(x-2)^2+(y-3)^2=1$ oriented counter clockwise.

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_D \left(2\frac{1}{1+(y/x)^2}\left(\frac{-y}{x^2}\right) - \left(1 - \frac{2y}{x^2+y^2}\right)\right) dA$$

$$= \iint_D \cancel{\frac{-2y}{x^2+y^2}} - 1 + \cancel{\frac{2y}{x^2+y^2}}\,dA$$

$$= -\pi. //$$

17.5:12

grad, applied to scalars

curl, applied to vectors

div, applied to vectors

a) no

b) yes

c) yes

d) yes

e) no

f) yes

g) yes

h) no

i) yes

j) no

k) no

l) yes

matter how we connect these points by path independence,

part a) take P_2 on the line $x-2y = 3\pi/2$
and P_1 on the line $x-2y = \pi/2$

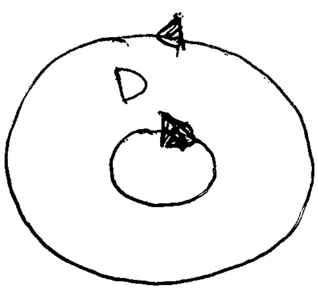
b) take P_2 on the line $x-2y=\pi/2$ & P_1 on the line $x-2y=0$.

17.4:8 compute

$$\int_C xe^{-2x}\,dx + (x^4+2x^2y^2)\,dy$$

where C is the boundary of the regions between the circles $x^2+y^2=1$ & $x^2+y^2=4$.

Soln:



$$\int_C xe^{-2x}\,dx + (x^4+2x^2y^2)\,dy$$

$$= \iint_D \left[\frac{\partial}{\partial x}(x^4+2x^2y^2) - \frac{\partial}{\partial y}\left[xe^{-2x}\right]\right] dA$$

$$= \iint_D (4x^3+4xy^2)\,dA = 0$$

By symmetry of region & the fact that x & x^3 are odd.