Final — Math 264 — Fall 2009

December 6, 2010

Remember to show your work. Do as many problems as you can.

1. Show that the equation $x^2 + y^2 + z^2 - 2x - 4y + 2z + 2 = 0$ represents a sphere. Find its radius and center.

2. (a) Find the vector projection of $\mathbf{b} = (1, 2, 2)$ onto $\mathbf{a} = (1, 1, 0)$.

   (b) Find the scalar projection of $\mathbf{b} = (1, 2, 2)$ onto $\mathbf{a} = (3, 4, 0)$.

3. (a) Find an equation of the plane parallel to $2x + 3y + 4z = 5$ which passes through the point $(1, 1, 1)$.

(b) Find an equation of the plane passing through the points $(1,0,0),(0,2,0)$ and $(0,0,3)$.

4. Find the parametrization of the line tangent to the curve $\vec{r}(t) = (t^2, e^{t-1}, t)$ at the point $P = (1,1,1)$.

5. Find an equation for the plane tangent to the surface $z^2 = ye^x$ at the point $(0,1,1)$.

6. Let $\vec{r}(t) = (t+1, t+2, t+3)$. Find a parametrization of a line parallel to $\vec{r}(t)$ which passes through point $\vec{r}_0 = (3,2,1)$.

7. Use the chain rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. Express your final answers in terms of s and t. $z = x^2 y$, $x(s,t) = st$, $y(s,t) = s^2$.

8. Draw the level sets of the following functions.

   (a) $f(x, y) = xy$

   (b) $g(x, y) = x^2 + y + 1$

9. Let $\mathbf{r}_1(t) = (t, t^2, t^3)$ and $\mathbf{r}_2(s) = (1 + s, 1 + s, 1 - s)$.

   (a) Find where the two curves intersect.

   (b) Find the angle of intersection.

10. Let $\mathbf{r}_1(t) = (a(t), b(t))$ and $\mathbf{r}_2(t) = (x(t), y(t))$. Prove

$$\frac{d}{dt}[\mathbf{r}_1(t) \cdot \mathbf{r}_2(t)] = \mathbf{r}_1'(t) \cdot \mathbf{r}_2(t) + \mathbf{r}_1(t) \cdot \mathbf{r}_2'(t).$$

11. Find the volume of the region bounded by $y^2 + z^2 = 9$, $x = 0$, $x = 3$ and above $z = 0$.

    (a) Setup the integral.

    (b) Compute the integral.

12. Find the volume under the graph of $f(x, y) = e^{-x^2-y^2}$ over the whole xy-plane.

    (a) Setup to integral.

    (b) Compute the integral.

13. Evaluate the line Integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (2xyz, x^2z, x^2y)$ and C is the straight line segment from the point $(0, 0, 0)$ to the point $(1, 1, 1)$ in two ways

   (a) By a direct computation (Apply the definition of the line integral).

   (b) By the Fundamental Theorem of Line Integrals (Compute the potential and apply the theorem).

14. Setup the integral $\iiint_T y dV$ where T is the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + y + z = 2$.

15. Setup the integral $\int_C ds$ where C is the curve parametrized by $\mathbf{r}(t) = (t, t^2, t^3)$ for $t \in [0, 1]$.

16. Evaluate the integral $\iiint_E yz dV$ where E lies above the $z = 0$ and below $z = y$ and inside the cylinder $x^2 + y^2 = 4$.

17. Let $f(x, y) = x^2 + 4x + y^2 + xy$.

    (a) Find the critical points of f.

    (b) Classify the critical points of f.

18. Evaluate $\int_C (2y + \sin(x))dx + (x + e^y)dy$ where C is the contour pictured below, oriented in the counter clockwise direction.

19. Evaluate $\iint_S (\nabla \times \vec{F}) \cdot \vec{N} dS$ where S is the part of the surface $x^2 + y^2 + z^4 = 1$ where $z \geq 0$ and $\vec{F} = (x, y, z^3)$. The surface S is oriented with an outward pointing normal.

20. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and S is a sphere of radius two centered at the origin. Also, let S have an outward pointing normal.