Wednesday September 14^{th} 16

$\boxed{\text{LAST TIME}}$

- The arc length of a curve from a point $\vec{\gamma}(t_0)$:

$$s = \int_{t_0}^{t} |\vec{\gamma}'(\tau)| \, d\tau = \int_0^t \sqrt{x'(\tau)^2 + y'(\tau)^2 + z'(\tau)^2} \, d\tau$$

$\boxed{\text{REMARKS ON HWs:}}$

1. $\cancel{\dfrac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} = 1}$ No!!

vector

scalar

2. Don't leave float expressions

3. Don't interchange "$\Rightarrow$" and "$=$"

$\Rightarrow$ means "implies"

4. Reasoning should go top to bottom.

example of "backwards" work:

$$\cancel{\begin{aligned} |\vec{a}\cdot\vec{b}| &\le |\vec{a}||\vec{b}| \\ \big||\vec{a}||\vec{b}|\cos(\theta)\big| &\le |\vec{a}||\vec{b}| \\ |\vec{a}||\vec{b}||\cos(\theta)| &\le |\vec{a}||\vec{b}| \\ |\cos(\theta)| &\le 1 \end{aligned}}$$

bad b/c started with the conclusion

better way:

$$\begin{aligned} |\vec{a}\cdot\vec{b}| &= \big||\vec{a}||\vec{b}|\cos(\theta)\big| \\ &= |\vec{a}||\vec{b}||\cos\theta| \\ &\le |\vec{a}||\vec{b}| \end{aligned}$$

$$\Rightarrow |\vec{a}\cdot\vec{b}| \le |\vec{a}||\vec{b}| \;//$$

HW QUESTIONS (In class)

#2 written

curvature is given by

$$K(t) = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,''(t)|^3}$$

If $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$, show,

$$K(t) = \frac{|x'(t)\,y''(t) - x''(t)\,y'(t)|}{\left(x''(t)^2 + y''(t)^2\right)^{3/2}}$$

The idea is to compute $|\vec{r}\,'(t) \times \vec{r}\,''(t)|$ and $|\vec{r}\,''(t)|$ and plug them into the formula.

For example:

$$|r''(t)| = \sqrt{x''(t)^2 + y''(t)^2}$$

Similarily, we compute $\vec{r}\,'(t) \times \vec{r}\,''(t)$ and plug both into The formula.

HW QUESTIONS cont...

#3 written : Reparametrization

reparametrize

$$\vec{\gamma}(t) = \left(\frac{2}{t^2+1} - 1, \frac{2t}{t^2+1} \right)$$

in terms of arc length from (1,0).

Steps for reparametrization:

1. Set up the integral

$$s = \int_{t_0}^{t} |\vec{\gamma}'(\tau)| \, d\tau \Big\} \text{ some fn in } t$$

[computing $\vec{\gamma}(t)$ is a little tricky, but it simplifies a lot]

2. Compute the integral

[Need to know trig integrals]

3. Solve for t in terms of s.

4. Plug this back into $\vec{\gamma}(t)$ to get the reparametrization

$$\vec{\rho}(s) = \vec{\gamma}(t(s))$$

In yesterday's example, $t(s) = \frac{s}{\sqrt{2}}$

$$\rho(s) = (\cos s/\sqrt{2}, \sin s/\sqrt{2}, s/\sqrt{2})$$

5. Use the $\frac{1}{2}$ angle formulas and $\tan(s/2)^2 + 1 = \sec(s/2)^2$ to simplify $\vec{\rho}(s)$ into something very simple

$s = \tan^{-1}(t)$

$t = \frac{\tan}{2}$