• Max & Mins of Functions:

10/03/16

<u>Definition</u>: A function f has a <u>local</u> max (or min) at the point (a,b) if $f(a,b) \geq f(x,y)$ for all (x,y) in a ball around (a,b).
($f(a,b) \leq (x,y)$ for local min)

- f has a <u>global</u> max or min if the inequality holds for all (x,y) in the domain of f

<u>Theorem:</u> If f has a local max or min at (a,b) and its first order derivatives (f_x & f_y) exist, then...

$$f_x(a,b) = f_y(a,b) = 0$$

<u>note:</u> f_x & f_y don't need to exist at a max or a min.

<u>Second Derivative Test</u>:

$$f_x(a,b) = f_y(a,b) = 0 \qquad D = \#$$

$$D = D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - (f_{xy}(a,b))^2$$

a.) if $D > 0$ and $f_{xx}(a,b) > 0$, then (a,b) is a local min.
b.) if $D > 0$ and $f_{xx}(a,b) < 0$, then (a,b) is a local max.
c.) if $D < 0$ then (a,b) is not a max or min, but a saddle point.

Example: Find all of the local max/mins of... 10/03/16

$$f(x,y) = x^2 + y^2 - 2x - 6y + 14$$

$$\frac{\partial x}{\partial F}(x,y) = F_x(x,y) = 2x - 2$$

$$\frac{\partial y}{\partial F}(x,y) = f_y(x,y) = 2y - 6$$

$f_x(x,y) = 0$ $\quad 2x - 2 = 0 \quad x = 1$

$f_y(x,y) = 0$ $\quad 2y - 6 = 0 \quad y = 3$

so $(1,3)$ is the critical point

$f_{xx}(x,y) = 2$ $\qquad D = D(x,y) = 2\cdot 2 - 0^2 = 4$

$f_{xy}(x,y) = 0$

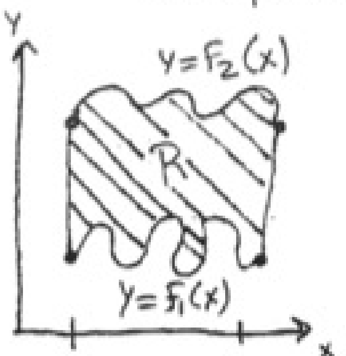
$f_{yy}(x,y) = 2$ $\qquad D = 4$ when $x=1$, $y=3$

$D > 0$, so we know $(1,3)$ is either a max or min

$f_{xx}(1,3) = 2 > 0$, so $(1,3)$ is a local min//

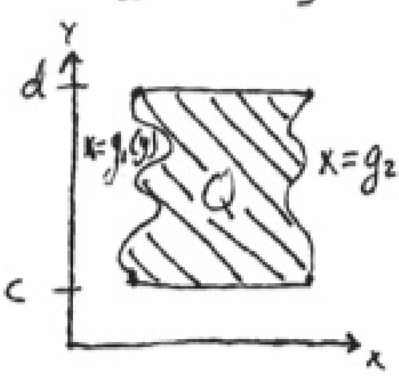
• Double Integrals: 10/07/16

- Help us compute volumes & areas we couldn't compute before



$$R = \{(x,y) \in \mathbb{R}^2 \mid a \le x \le b\}$$

$$A_R = \iint_R da = \int_a^b \left[\int_{f_1(x)}^{f_2(x)} dy\right] dx$$



$$\text{Area } Q = \iint dA$$

$$= \int_c^d \int_{g_1(y)}^{g_2(y)} dx\, dy$$