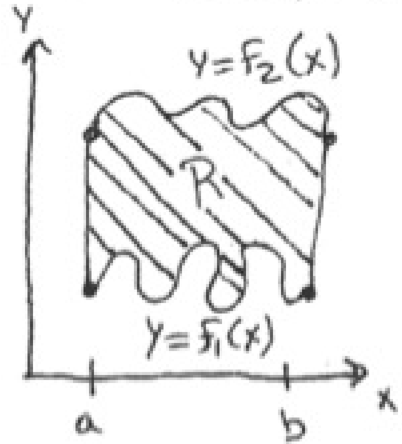
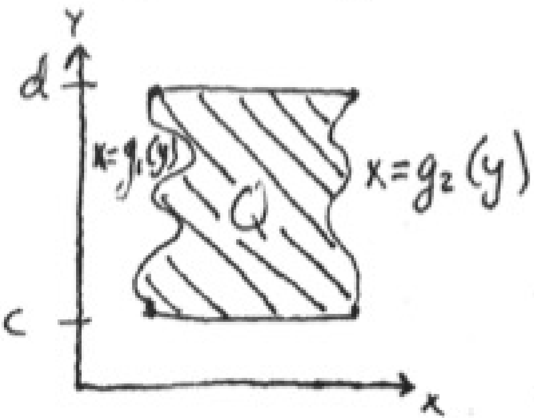
· DOUBLE INTEGRALS: 10/07/16

- Help us compute volumes & areas we couldn't compute before



$$R = \{(x,y) \in \mathbb{R}^2 \mid a \le x \le b\}$$

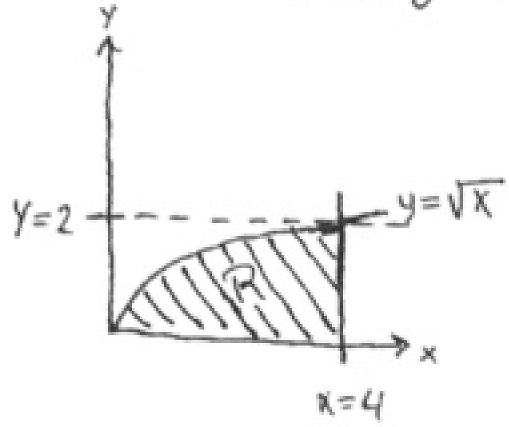
$$A_R = \iint_R da = \int_a^b \left[\int_{f_1(x)}^{f_2(x)} dy\right] dx$$



$$\text{Area } Q = \iint dA$$

$$= \int_c^d \int_{g_1(y)}^{g_2(y)} dx\,dy$$

Example: Find the area of the following region using double integration 10/7/16



$$A_R = \int_0^4 \int_0^{\sqrt{x}} dy\, dx$$

$$A = \int_0^4 (\sqrt{x} - 0)\, dx$$

$$A = \int_0^4 x^{\frac{1}{2}}\, dx = \left.\frac{x^{3/2}}{3/2}\right|_{x=0}^{x=4} = \frac{2}{3}\left(4^{3/2}\right) = \boxed{\frac{16}{3}}$$

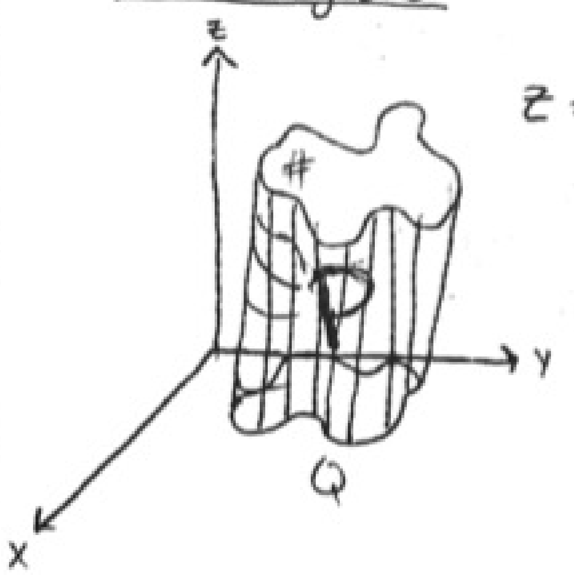
- We can do this another way, by integrating with respect to x-first

$$x \in [y^2, 4] \qquad y \in [0, 2]$$

$$A = \int_{y^2}^4 \int_0^2 dx\, dy = \int_0^2 (4-y)^2\, dy$$

$$= 4y - \left.\frac{y^3}{3}\right|_{y=0}^{y=2} = 4(2) - \frac{(2)^3}{3} = \boxed{\frac{16}{3}}$$

- Double integrals can also be used to find volumes of 3D regions:

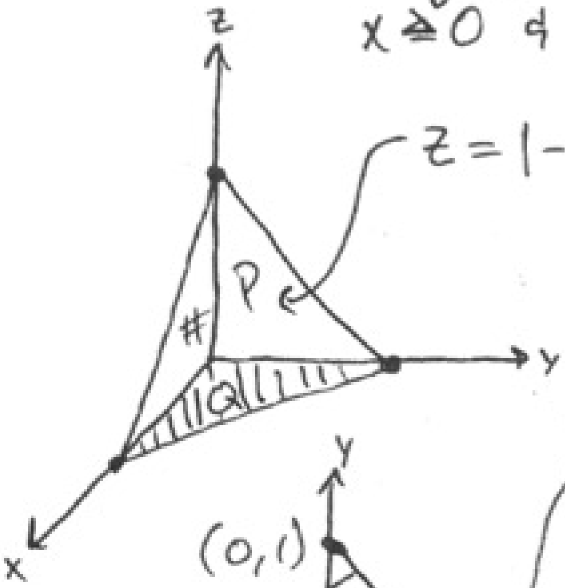


$z = f(x, y)$ Q is the region this is over in $\mathbb{R}^2$

$$\text{Vol}(P) = \iint_Q f(x,y)\, dA$$

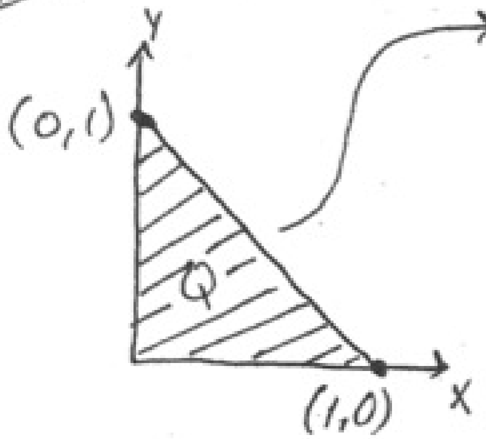
Example: Find the volume under the plane 10/07/16

$x+y=1$ and above the xy plane with $x \geq 0$ & $y \geq 0$



$z = 1 - x + y = P$

We know $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ are points on the plane.



$$\iint_Q (1-x+y)\,dA$$

$$Q = \{(x,y) : 0 \leq y \leq 1-x \ \&\ 0 \leq x \leq 1\}$$

$$\begin{pmatrix}\text{Volume of} \\ \text{Region}\end{pmatrix} = \iint_Q (1-x+y)\,dA \longrightarrow \text{doing } x \text{ first, we get}$$

$$\int_0^{1-y}\int_0^1 (1-(x+y))\,dx\,dy = \int_0^1 \left[\int_0^{1-y} (1-(x+y))\,dx\right] dy$$

$$= \int_0^1 \left[\left((1-y) - \frac{(1-y)^2}{2} - (1-y)y\right) - \cancel{\left(0 - \frac{0^2}{2} - 0y\right)}\right] dy$$

$$= \int_0^1 \left[(1-y)^2\left(1-\frac{1}{2}\right)\right] dy = \frac{1}{2}\int_0^1 (1-y)^2\,dy$$

$$= \frac{1}{2}\left(-\frac{(1-y)^3}{3}\right)\Bigg|_{y=0}^{y=1} = \frac{1}{2}\left(0 - \left(-\frac{1}{3}\right)\right) = \boxed{\frac{1}{6}}$$