- <u>DOUBLE INTEGRAL TRICKS & POLAR COORDINATES:</u> 10/12/16

  - <u>Fubini's Theorem</u>: you can <u>flip</u> constant bounds of integration.

$$\int_a^b\int_c^d f(x,y)\,dx,dy = \int_c^d\int_a^b f(x,y)\,dx\,dy$$

- Separation of variables 10/12/16

$$\int_c^d\int_a^b f(x)g(y)\,dx\,dy = \left(\int_a^b f(x)dx\right)\left(\int_c^d g(y)\,dy\right)$$

WARNING! these tricks only work for constant bounds of integration. Usually the inner bounds of integration are variable.

- Example$_1$: $\int_0^1\int_0^2 (x+y)\,dx\,dy = \int_0^1 \left(\frac{x^2}{2}+yx\right)\Big|_{x=0}^{x=2} dy$

$$= \int_0^1 \left(\frac{4}{2}+2y\right)dy = \left(2y + 2\frac{y^2}{2}\right)\Big|_{y=0}^{y=1} = 2+1 = 3$$

- Example$_2$: $\int_0^3\int_1^2 x^2 y\,dy\,dx = \left(\int_0^3 x^2 dx\right)\left(\int_1^2 y\,dy\right)$

$$= \left(\frac{x^3}{3}\Big|_{x=0}^{x=3}\right)\left(\frac{y^2}{2}\Big|_{y=1}^{y=2}\right) = \frac{27}{3}\left(\frac{4}{2}-\frac{1}{2}\right) = \boxed{\frac{27}{2}}$$

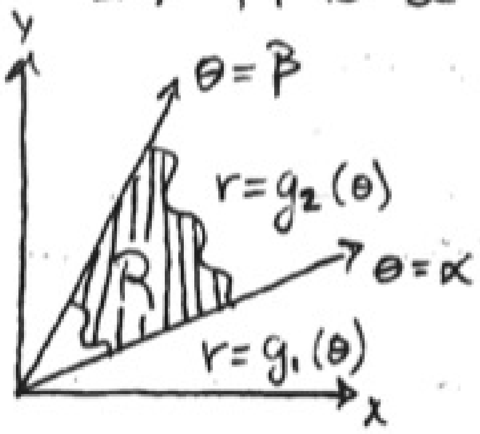
- POLAR COORDINATES:

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

$(r,\theta) \longrightarrow$ Polar coordinates

- Theorem: (change of variables for polar coordinates)

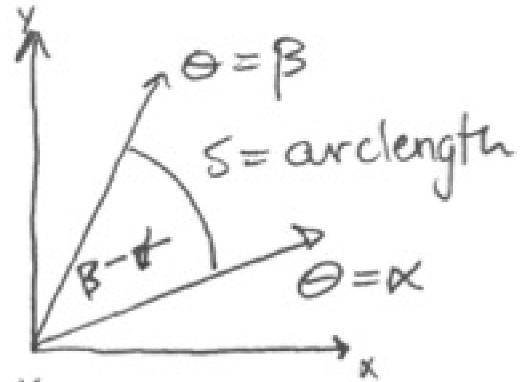
If R is a region in $\mathbb{R}^2$, describe the following Picture



$$\iint_R f(x,y)\,dA = \int_\alpha^\beta\int_{g_1(\theta)}^{g_2(\theta)} f(r\cos(\theta), r\sin\theta)\,r\,dr\,d\theta$$

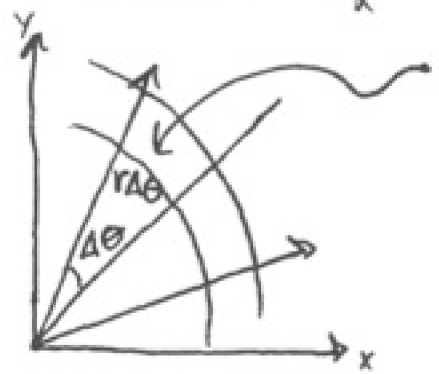
- Simplifies cartesian integrals
- integration over new regions

Recall: 10/12/16



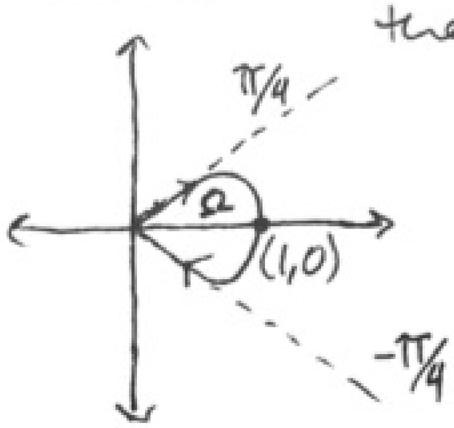
$$(\text{Area}) = \frac{(\beta-\alpha)}{2} R^2$$

$$s = (\beta - \alpha) R$$



$$r\,dr\,d\theta = dA$$

- Example: Find the area of the region enclosed by the curve $r=\cos(2\theta)$ when $x \geq 0$



$x = r\cos\theta$, $y = r\sin\theta$

$$\cos(2(-\pi/4)) = 0$$
$$\cos(2(\pi/4)) = 0$$
$$\cos(2(0)) = 1$$

note: $0 \leq r \leq \cos(2\theta)$

$$-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$$

$$\iint_\Omega dA = \int_{-\pi/4}^{\pi/4}\int_0^{\cos(2\theta)} r\,dr\,d\theta$$

$$\cancel{= \int_{-\pi/4}^{\pi/4}\int_0^{\cos(2\theta)}}$$

$$\cancel{= \int_{-\pi/4}^{\pi/4}\int_0^{\cos(2\theta)}} = \int_{-\pi/4}^{\pi/4}\left[\frac{r^2}{2}\Big|_{r=0}^{r=\cos(2\theta)}\right] d\theta$$

$$= \int_{\pi/4}^{\pi/4} \frac{\cos(2\theta)^2}{2}\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(u)^2}{2}\,\frac{du}{2}$$

SIDE WORK:	Bounds:
$u = 2\theta$	$\theta = \pi/4 \Rightarrow u = 2(\frac{\pi}{4}) = \frac{\pi}{2}$
$du = 2d\theta$	$\theta = -\frac{\pi}{4} \Rightarrow -\frac{\pi}{2}$
$\frac{du}{2} = d\theta$	

$$= \frac{1}{4}\int_{-\pi/2}^{\pi/2} \cos(u)^2\,du$$

$$= \frac{1}{4}\left[2\int_0^{\pi/2} \cos(u)^2\,du\right]$$

10/12/16

$= \int_0^{\pi/2} \cos(u)^2 \, du = \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$ = area of