MATH [redacted] 121 - Solutions to HW01 (written part) - DUPUY

1)

1.a) (5 pts)
Show that $\vec{a}$ & $\vec{a} \times \vec{b}$ are orthogonal.

sol,

Let $\vec{a} = (a_1, a_2, a_3)$. Let $\vec{b} = (b_1, b_2, b_3)$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = (a_1, a_2, a_3) \cdot (a_2b_3 - a_3b_2, -a_1b_3 + a_3b_1, a_1b_2 - b_1a_2)$$

$$= a_1(a_2b_3 - a_3b_2) - a_2(a_1b_3 - a_3b_1) + a_3(a_1b_2 - a_2b_1)$$

↑ everything cancels

$$= 0. \;//$$

1.b) (5 pts)

Prove that $|\vec{a}\cdot\vec{b}| \leq |\vec{a}||\vec{b}|$.

proof

$$|\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}|\cos(\theta)|$$

$$= |\vec{a}|\,|\vec{b}|\,|\cos(\theta)|$$

$$\leq |\vec{a}||\vec{b}|$$

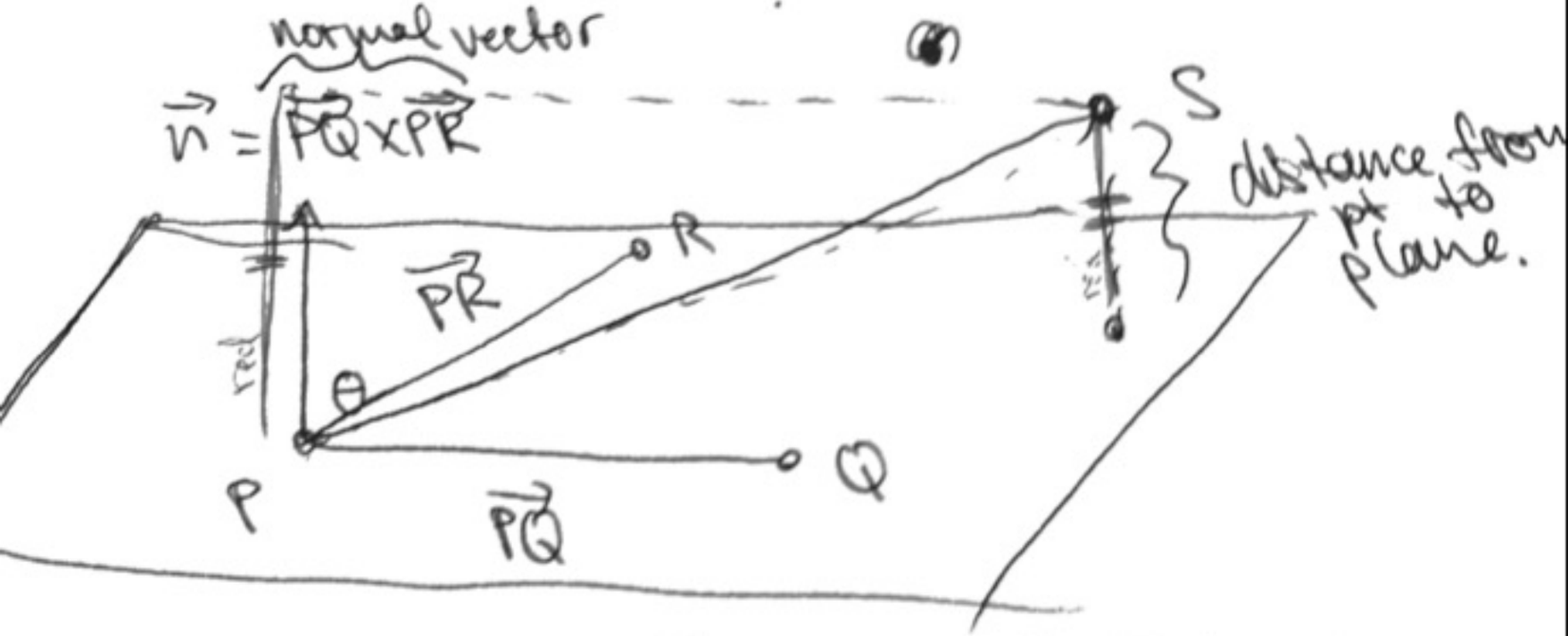
The last line follows from the fact that

$|\cos(\theta)| \leq 1$. //

2) (10 pts)
Show the distance from the plane spanned by points P, Q & R to another point S is

$$\left| \vec{PS} \cdot \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} \right|.$$

Solution.
We first draw a picture



One can see that the two labelled red distances are the same and from trig that this distance is

$$\left| |\vec{PS}| \cos(\theta) \right| \qquad (*)$$

where θ is the angle between $\vec{PS}$ & the normal vector $\vec{PQ} \times \vec{PR}$. Using the

Solution to 2 cont...:

Formula relating angles and dot products we found

$$\vec{PS} \cdot (\vec{PQ} \times \vec{PR}) = |\vec{PS}|\,|\vec{PQ} \times \vec{PR}|\cos(\theta), \quad (*)$$

We solve this equation for $\cos(\theta)$ and plug it back into $(*)$ to get our result:

$$\cos(\theta) = \frac{\vec{PS} \cdot (\vec{PQ} \times \vec{PR})}{|\vec{PS}|\,|\vec{PQ} \times \vec{PR}|}$$

$$\Rightarrow (\text{distance}) = \left| |\vec{PS}| \cos(\theta) \right|$$

$$= \left| \cancel{|\vec{PS}|} \frac{\vec{PS} \cdot (\vec{PQ} \times \vec{PR})}{\cancel{|\vec{PS}|}\,|\vec{PQ} \times \vec{PR}|} \right|$$

$$= \left| \vec{PS} \cdot \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} \right| . \quad //$$