Homework 02 - Solutions - Dupuy

Problem 1:

1a) We plug the line

$$x = a + t$$
$$y = b + t$$
$$z = c + 2(b-a)t$$

into the hyperbolic paraboloid $z = y^2 - x^2$.

$$c + 2(b-a)t \overset{?}{=} (b+t)^2 - (a+t)^2$$

$$\iff c + 2(b-a)t = b^2 + \underline{\underline{2bt}} + \cancel{t^2} - a^2 - \underline{\underline{2at}} - \cancel{t^2}$$

$$\iff c = b^2 - a^2.$$

The last eqn is satisfied because (a,b,c) is a point on the curve.

1b)

$(a,b,c) = (1,1,0)$

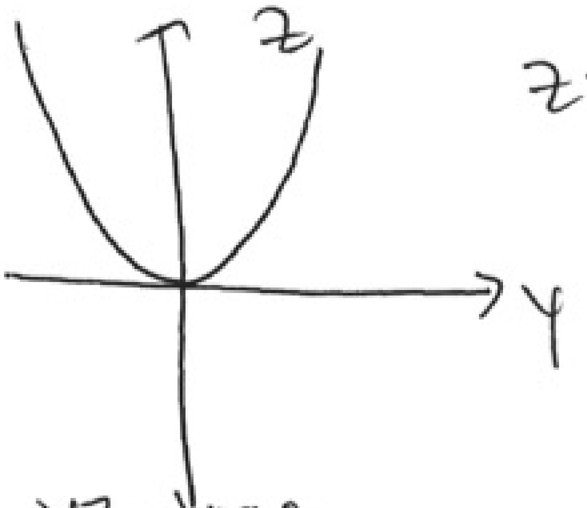
$$\Rightarrow \begin{cases} x = 1+t \\ y = 1+t \\ z = 0 \end{cases}$$

this is just the line $y=x$ in the $z=0$ plane.

This is contained in the ~~parabola~~ hyperboloid.

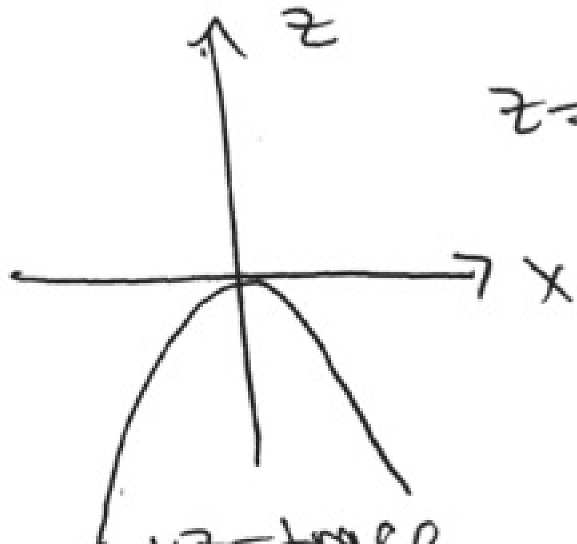
[redacted]

Here are the traces:



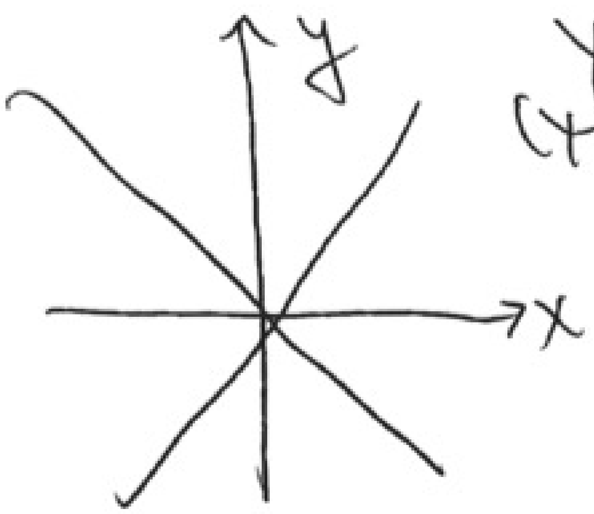
$z = y^2$

yz-trace



$z = -x^2$

xz-trace



$y^2 - x^2 = 0$

$(y-x)(x+y) =$

xy-trace

Problem 2:

let $\vec{u} = (u_1, u_2, u_3)$, $\vec{v} = (v_1, v_2, v_3)$.

$$\vec{u} \times \vec{v} = (u_2 v_3 - v_2 u_3, -(u_1 v_3 - u_3 v_1), u_1 v_2 - u_2 v_1)$$

$$\frac{d}{dt}[\vec{u} \times \vec{v}] = [u_2' v_3 - u_2 v_3' \pm v_2' u_3 - v_2 u_3') \hat{i}$$
$$- [u_1' v_3 + u_1 v_3' - u_3' v_1 - u_3 v_1'] \hat{j}$$
$$+ [u_1' v_2 + u_1 v_2' - u_2' v_1 - u_2 v_1'] \hat{k}$$

$$= (u_2' v_3 - v_2 u_3') \hat{i} \pm (u_1' v_3 - u_3' v_1) \hat{j}$$
$$+ (u_1' v_2 \pm u_2' v_1) \hat{k}$$
$$+ (u_2 v_3' - v_2' u_3) \hat{i} - (u_1 v_3' - u_3 v_1') \hat{j}$$
$$+ (u_1 v_2' - u_2 v_1') \hat{k}$$

$$= \vec{u}' \times \vec{v} + \vec{u} \times \vec{v}'. //$$

Problem 3:

$$\frac{d}{dt}\left[\vec{l}(t)\right] = \frac{d}{dt}\left[\vec{r}(t) \times \vec{p}(t)\right]$$

$$= \frac{d}{dt}\left[\vec{r}(t)\right] \times \vec{p}(t) + \vec{r}(t) \times \frac{d}{dt}\left[\vec{p}(t)\right]$$

$$= \vec{v}(t) \times m\vec{v}(t) + \vec{r}(t) \times \vec{F}(t)$$

$$= m\left(\cancel{\vec{v}(t) \times \vec{v}(t)}\right)^{0} + \vec{r}(t) \times \vec{F}(t)$$

$$= \vec{\tau}(t).$$

We used:

- product rule for cross products.
- derivative of position is velocity
- derivative of momentum is force
- the cross product of two parallel vect
  is zero.

Problem 4 had a typo so I am not grading it.