Homework 04 - solutions - Dupuy

Problem 1: Show

$$u(x,t) = f(x-ct) + g(x+ct)$$

is a solution of

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0.$$

Soln.

$$\frac{\partial^2}{\partial t^2}\left[f(x-ct) + g(x+ct)\right] - c^2 \frac{\partial^2}{\partial x^2}\left[\cancel{g(x+ct)}\; f(x-ct) + g(x+ct)\right]$$

$$= \cancel{(-c)^2 f''(x-ct)} + \cancel{c^2 g''(x+ct)}$$

$$- \cancel{c^2}\left[\cancel{f''(x-ct)} \; \cancel{c^2} + \cancel{g''(x+ct)}\right]$$

$= 0.$ //

Problem 2. $PV = nRT$

Show

(a) $\frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = -1$

(b) $T \frac{\partial P}{\partial T} \frac{\partial V}{\partial T} = nR$

Solution.

(a) $P = \frac{nRT}{V}$, $V = \frac{nRT}{P}$, $T = \frac{PV}{nR}$.

$$\Rightarrow \frac{\partial P}{\partial V} = \frac{-nRT}{V^2}$$

$$\frac{\partial V}{\partial T} = \frac{nR}{P}$$

$$\frac{\partial T}{\partial P} = \frac{V}{nR}$$

$$\Rightarrow \frac{\partial P}{\partial V} \frac{\partial V}{\partial T} \frac{\partial T}{\partial P} = \left(\frac{-nRT}{V^2}\right)\left(\frac{\cancel{nR}}{P}\right)\left(\frac{\cancel{V}}{\cancel{nR}}\right)$$

$$= \frac{-nRP}{PV} = -1 \quad (PV = nRT)$$

(b) $\frac{\partial P}{\partial T} = \frac{nR}{V}, \quad \frac{\partial V}{\partial T} = \frac{nR}{P}$

$$\Rightarrow T \frac{\partial P}{\partial T}\frac{\partial V}{\partial T} = T\left(\frac{nR}{V}\right)\left(\frac{nR}{P}\right)$$

$$= T\frac{(nR)^2}{PV}$$

$$= \frac{\cancel{T}(nR)^{\cancel{2}}}{\cancel{nRT}}$$

$$= nR. \,//$$

Problem 3

$$u = e^{a_1x_1 + \cdots + a_nx_n}$$

show $\displaystyle\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = u.$ $\left[\text{assume } a_1^2 + \cdots + a_n^2 = 1\right]$

Solution:

$$\frac{\partial^2 u}{\partial x_i^2} = a_i^2 e^{a_1x_1 + \cdots + a_nx_n}$$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = \sum_{i=1}^{n} a_i^2 e^{a_1x_1 + \cdots + a_nx_n}$$

$$= e^{a_1x_1 + \cdots + a_nx_n} \sum_{i=1}^{n} a_i^2$$

$$= u(1)$$

$$= u. //$$