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Friday 9/16/16

Reminders:

- Midterm next Friday (no calculators) → Everything Ch 12, 13, 14.1
  ↳ Review Next week
- Webpage updates:
  - ↳ Piazza (link on course webpage)
  - ↳ course notes will be posted
  - ↳ link to sage (open source mathematica)

LAST TIME:

- Mathematical writing
- Homework problem session

TODAY:

- Partial derivatives
- Contour plots/ level sets
- Sage (if time)

Partial Derivatives → If $\underbrace{f = f(x, y, z)}$

"f is a function of 3 variables"

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y, z) - f(x, y, z)}{\Delta x} \Big\}$$ partial derivative of f with respect to x

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y, z) - f(x, y, z)}{\Delta x} \Big\}$$ " " y

$$\frac{\partial f}{\partial z} = \lim_{\Delta z \to 0} \frac{f(x, y, z+\Delta z) - f(x, y, z)}{\Delta x} \Big\}$$ " " z

What this does:

$\frac{\partial}{\partial x}$ = "take a derivative with respect to x and treat all other variables as constants"

Similarily w/ $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$

Examples:

$\rightarrow \frac{\partial}{\partial x}\left[x^2+y\right] = 2x+0$

$\rightarrow \frac{\partial}{\partial x}\left[e^y\right] = 0$

$\rightarrow \frac{\partial}{\partial x}\left[xy\right] = y$

$\rightarrow \frac{\partial}{\partial y}\left[xy\right] = x$

$\rightarrow \frac{\partial}{\partial z}\left[xy\right] = 0$

$$\rightarrow \frac{\partial}{\partial s}\left[s^2 e^{st} + f(s)\right]$$

$$= \frac{\partial}{\partial s}\left[s^2 e^{st}\right] + \frac{\partial}{\partial s}\left[f(s)\right]$$

$$= 2se^{st} + s^2 e^{st}\frac{\partial}{\partial s}[st] + f'(s)$$

$$= (2s+s^2t)e^{st} + f'(s)$$

$\boxed{\text{Notations for partial derivatives}}$

If $f = f(x,y,z)$, then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}(x,y,z) = f_x = f_x(x,y,z)$$

$\boxed{\text{We can iterate partial derivatives:}}$

Examples:

$$\rightarrow \frac{\partial^2}{\partial x \partial y}\left[x^2y+y\right] = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}\left[x^2y+y\right]\right] = \frac{\partial}{\partial x}\left[x^2+1\right] = 2x$$

$$\rightarrow \frac{\partial^2}{\partial y \partial x}\left[x^2y+y\right] = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}\left[x^2y+y\right]\right] = \frac{\partial}{\partial y}\left[2xy+0\right] = 2x$$

$\boxed{\text{REMARK}}$ It is not a coincidence that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ in this example. This will be true for differentiable functions.

- warning! → Differentiable does ~~not~~ mean that the partial derivatives exist
  → Differentiable means that one can find a linear approximation.
  ↳ He will explain this later...

CONTOUR PLOTS/LEVEL SETS

Defn → let $f(x,y,z)$ be a fn. A level set or contour of $f(x,y,z)$ is a set of the form

$$\{(x,y,z) : f(x,y,z) = c\}$$

for some constant c.

Remarks:

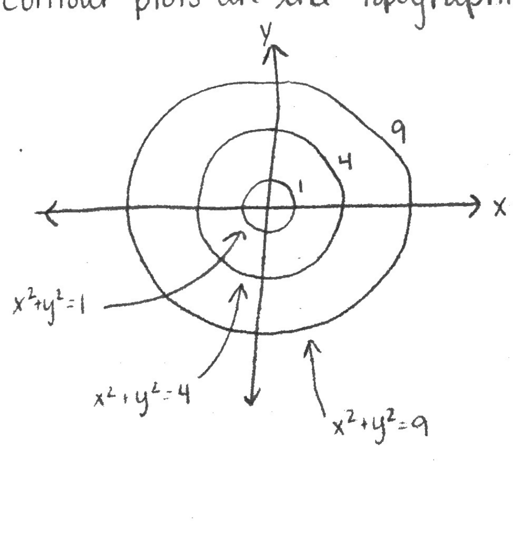
- one can make a similar 2D defn
- A level set is just a set of pts where the fn stays level

Example: Plot the level sets of

$$f(x,y) = x^2+y^2$$

soln: $\{(x,y): x^2+y^2 = c\}$ = (level set for value c)

Contour plots are like topographical maps



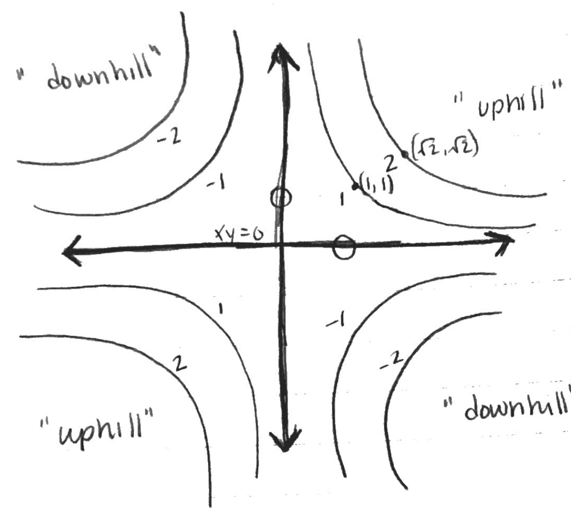
Example: Plot the level sets of

$$g(x,y) = xy$$

soln:

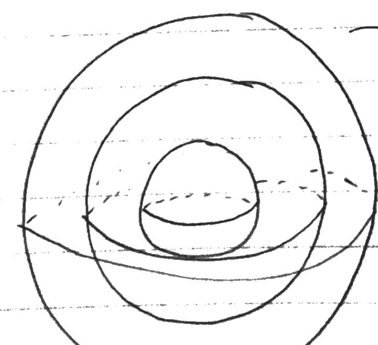
look at $xy = c$

$\Rightarrow$ $y = c/x$ } graph these fns for varying c



Example: what are the level sets of $g(x,y,z) = x^2+y^2+z^2$?

Soln: concentric spheres



" 3D contour plot"

(there are graphing programs to draw these)