Monday 9/19/2016

HOMEWORK 4 CORRECTIONS

#1) → should read

$$u_{xx} = c^2 u_{tt}$$

#3) $\frac{\partial^2 u}{\partial x_1^2} + \dots + \frac{\partial^2 u}{\partial x_n^2} = u$

~~$\frac{\partial^2 u}{\partial x_1^2} + \dots + \frac{\partial^2 u}{\partial x_n^2} = u$~~

~~$u_{xx} - c u_{tt} = 0$~~

(corrected hw will be posted)

SAGE

↳ worksheets will be posted

LAST TIME

↳ Partial Derivatives

$$\frac{\partial}{\partial x}\left[e^{xy} z + z^2 y\right] = yze^{xy}$$

↳ contour plots / Level Set Plots

$$\begin{pmatrix}\text{level set} \\ \text{of } f(x,y)\end{pmatrix} = \begin{pmatrix}\text{collection of pts where } f(x,y) \\ \text{is constant}\end{pmatrix}$$

example: level sets of $f(x,y) = x^2 + y^2$ are circles

TODAY: Limits and Symbolic Computing

$\lim_{(x,y)\to(a,b)} f(x,y) = L$

"The limit of $f(x,y)$ as (x,y) approaches (a,b)

meaning: As you approach (a,b) from <u>any</u> direction, $f(x,y)$ approaches L.

Big change in calc 3: We can approach from multiple directions.

<u>Example problem:</u> Compute the limits if they exist. If not, state why.

a) $\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2}$

b) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

c) $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$

RECALL → $\frac{xy}{x^2+y^2}$ & $\frac{x^2y}{x^2+y^2}$ were plotted in our sage sessions.

Soln:

a) Nothing is going on here.

$$\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{(1)(1)}{(1)^2+(1)^2} = \boxed{\frac{1}{2}}$$ ← This is the limit

b) When you plug in, you get $\frac{0}{0}$ so you need to do more work

TRICK → convert to polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{r\to 0} \frac{r\cos\theta\, r\sin\theta}{r^2} = \lim_{r\to 0} \cos\theta\sin\theta$$

The limit does not exist

We will explain later ...